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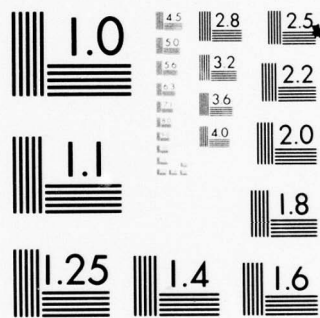
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LEVEL II

CSL COORDINATED SCIENCE LABORATORY

DECISION AND CONTROL LABORATORY

FEEDBACK CONTROLLED AIRCRAFT SENSITIVITY TO PARAMETER VARIATIONS

ROBERT LEA JACKSON

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FEEDBACK CONTROLLED AIRCRAFT SENSITIVITY

TO PARAMETER VARIATIONS

BY

ROBERT LEA JACKSON

B.S., Purdue University, 1975

THESIS

Submitted in partial fulfillment of the requirements
for the degree of Master of Science in Electrical Engineering
in the Graduate College of the
University of Illinois at Urbana-Champaign, 1977

Thesis Adviser: Professor J. B. Cruz, Jr.

Urbana, Illinois

FEEDBACK CONTROLLED AIRCRAFT SENSITIVITY
TO PARAMETER VARIATIONS

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Abstract

This report is concerned with a sensitivity study of a
flight control design using optimal control theory and observer theory.
A simplified model of an airplane's vertical motion is used. Hybrid
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control design might be preferable.

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Systems group.

TABLE OF CONTENTS

CHAPTER 1

INTRODUCTION

The need for more advanced optimal control techniques has become apparent in recent years. With the advent and perfection of minicomputers (microprocessors) digital aircraft control has come of age. Need for control optimization can be applied to commercial aircraft in the form of fuel economizing and increased desire for instrument landings, as well as its current application to defense and aerospace needs. One major advantage of this control technique is the reduction or elimination of the need for a pilot, for certain tasks, thus reducing the possibility of human errors while retaining the effects of human judgement.

In order to apply optimal control techniques, certain assumptions and simplifications are usually made. One such simplification involves the linearization of the plant equations around one flight condition. The question then arises as to how this will affect the response of the system at other flight conditions. Using sensitivity analysis this thesis examines these effects along with the effects of using an observer to estimate certain unavailable states.

The method used here, begins by deriving the equations of motion of an airplane. These equations are used to simulate aircraft flight. Using this model as the plant to be controlled, the nonlinear equations are linearized around a specified flight condition. With these linearized matrices a PID control is then designed by using the minimum principle. For the case where certain states are not available, a low order observer is then designed. This controller can then be applied to the nonlinear system and tested for its acceptability.

The objective of this thesis, therefore, is to study the practicality of this control under simulated flight conditions. This is accomplished first by determining how sensitive the closed loop system is to various parameter variations. Then the effects of using an observer are examined. Finally, the controls are tested to see how well they perform in an actual real time application.

The procedure followed in this thesis is typical of the method used for design of a linear control for application to a nonlinear plant. The derivations are general enough that this same procedure could be used to control a variety of other systems.

CHAPTER 2

AIRCRAFT MODELLING

In order to proceed with precise and efficient controlling of a system, an accurate model of the system must first be found. In this thesis a simplified model of an airplane's vertical equations of motion is used. Theoretically, though, any system could be used and controlled using the techniques of the following chapters (e.g. lateral equations of motion).

2.1 EQUATIONS OF MOTION

The equations necessary for this model can be derived using simple dynamics while making certain reasonable assumptions about the aircraft (ignoring aeroelasticity, etc.). In general an airplane can be assumed to have the configuration as shown in Fig.1 where the symbols refer to the quantities as given in Table 1.(Ref. 4,6,7).

The X-axis is usually (by definition) such that the angle of attack (α) is small. Therefore small angle approximations can be made. This leads to the following.

$$\sin(\alpha) = \alpha$$

$$\cos(\alpha) = 1$$

$$u = V\cos(\alpha) = V$$
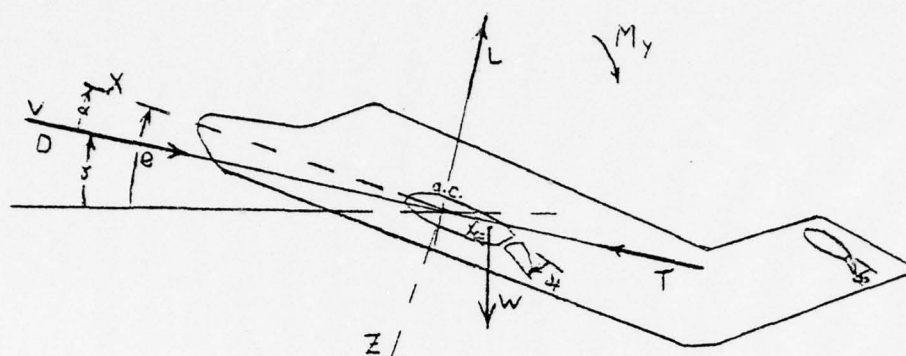
Figure 1. Airplane Configuration

Table 1

Symbols

α: angle of attack

θ: pitch angle

γ: flight path angle

M: mass of the aircraft

V: velocity

H: altitude

W: weight of the aircraft

I_{yy}: moment of inertia

X_{cg}: center of gravity

S: wing surface area

ρ: air density

c: chord length

X: body axis

Z: vertical normal to body axis

$$\dot{u}=\dot{V}\cos(\alpha)-V\sin(\alpha)\dot{\alpha}=\dot{V}-Va\dot{\alpha}$$

$$w=V\sin(\alpha)=v\alpha$$

$$\dot{w}=\dot{V}\sin(\alpha)+V\cos(\alpha)\dot{\alpha}=\dot{V}a+V\dot{\alpha}=V\dot{\alpha}$$

$$\sin(\gamma)=\sin(\gamma-\alpha)=\sin(\gamma)\cos(\alpha)-\cos(\gamma)\sin(\alpha)$$
$$=\sin(\gamma)-\alpha\cos(\gamma)$$

Summing the forces in the X-direction:

$$0=-M\dot{V}+T-D-W\sin(\gamma)+L\sin(\alpha)-(D-T)\cos(\alpha)$$

Now using these approximations:

$$0 = -M\dot{V} + T - D - W(\sin(\gamma) - \alpha\cos(\gamma))$$
$$+ (L - W\cos(\gamma) - MV(\dot{\theta}-\dot{\alpha}))$$

So, using 2.2) from below:

2.1) $0 = -M\dot{V} + T - D - W\sin(\gamma)$

Summing the forces in the Z direction:

$$0 = -M(\dot{w} - uq) + W\cos(\gamma) - L\cos(\alpha) - (D - T)\sin(\alpha)$$

Drag and thrust terms are generally smaller than the lift and weight terms, and about equal to each other. Using this plus the fact that they are multiplied by a small angle, α, the last term above is considered negligible. So this simplifies to:

2.2) $0 = L - W\cos(\gamma) - MV(\dot{\theta}-\dot{\alpha})$

Summing the moment in the X-Z plane:

2.3) $I\ddot{\theta} = M_y$

Summarizing these equations:

2.4) $\dot{V} = \frac{1}{M}(T - D - W\sin(\gamma))$

$$\dot{\alpha} = \dot{\theta} - \frac{1}{MV}(L - W\cos(\gamma))$$

2.5) $\dot{H} = V\sin(\gamma)$

$\ddot{\theta} = M_y / I_{yy}$

$L = \rho \frac{1}{2} V^2 S C_l$

$D = \frac{1}{2}\rho V^2 S C_d$

$M_y = \frac{1}{2}\rho V^2 S c C_m$

Where C_l, C_m, and C_d depend on wing plan form used and placement of the wing (and sometimes placement of the engines). All the coefficients in these equations can be found for any size airplane using the specified configuration and by looking up the wing specifications in a book listing wing characteristics. These equations are generally simplified for Mach numbers less than 1.0 by:

$C_l = C_{lo} + C_{la} + C_{lf}\delta_f$

$C_d = C_{do} + C_{dcl} + C_{df}\delta_f$

$C_m = C_{mo} + C_{mcl}C_l + C_{me}\ e + C_{mf}\delta_f$

$$- \frac{c}{2V}(\dot{\alpha} + q)$$

Any airplane now can theoretically be simulated, perhaps with minor modifications due to engine placement, tail configuration or Mach number. For simplicity the coefficients of the GAT II simulator as described in Daly's thesis (Ref. 4) are used with minor revisions.

Thrust is a more complicated subject. It is highly dependent on Mach number , altitude, and the type of engine used (turboprop, turbofan, propeller, etc.). In general there are no easily found formulas for thrust. Again for

simplicity, and due to a limited knowledge of this subject, the thrust formulation (propeller) used in Daly's thesis was adopted, which is:

$$Map = C_{po} + C_{ph}H + C_{pn}N + C_{pnt}N\delta_t$$

$$Bhp = C_{bo} + C_{bn}N + C_{bp}Map + C_{bh}H$$

$$T = NeBhp(C_{to} + C_{tv}V + C_{th}H + C_{tvh}VH)$$

where the abbreviations are as listed in Table 2.

Table 2

Thrust abbreviations

Map: manifold pressure

N: RPM

Bhp: brake horsepower

T: thrust

δ_t: throttle

Ne: number of engines

Throughout this work just one flight condition was examined, although any reasonable conditions could have been examined. The values that are used are shown in Table 3.

Combining all these equations yields the fifth order nonlinear system below, where the states x_1-x_5 and controls u_1-u_3 represent the quantities listed in Table 4.

2.6) $$\dot{x}_1 = x_4 - \frac{1}{Mx_2}[\frac{1}{2}\rho x_2^2 S(C_{1o} + C_{1a}x_1 + C_{1f}u_2)$$
$$- Wcos(x_3 - x_1)]$$

Table 3

Coefficients and flight conditions used

$C_{lo} = 0.0765$

$C_{la} = 4.62$

$C_{lf} = 0.365$

$C_{do} = 0.026$

$C_{dcl} = 0.062$

$C_{df} = 0.021$

$C_{mo} = 0.1$

$C_{mcl} = -0.0529 + \frac{X}{c}cg$

$C_{me} = -0.0354$

$C_{mf} = -0.0368$

$C_{bhp} = 2.11$

$C_{po} = 29.92$

$C_{ph} = 0.0009$

$C_{pn} = 0.00076$

$C_{pnt} = 0.0165$

$C_{bo} = -352.3$

$C_{bn} = 0.1155$

$C_{bp} = 10.8$

$C_{bh} = 0.0025$

$C_{to} = 3.5$

$C_{tv} = -0.00642$

$C_{th} = -4.73(10)^{-5}$

$C_{tvh} = 8.7(10)^{-8}$

$N = 2500$ rpm

$\rho = 0.004842$ slugs/ft^3

$S = 180.$ ft^2

$X_{cg} = 0.2$ ft

$c = 5$ ft

$W = 4000.$ lbs

$I_{yy} = 2050.$ slugs ft^2

$V_o = 190.66$ ft/sec

$H_o = 2000.$ ft

$\theta_c = \dot{\theta}_o = \alpha_c = 0.$

2.7) $\quad \dot{x}_2 = \frac{1}{M}(-W\sin(x_3 - x_1) - \frac{1}{2}\rho x_2^2 S(C_{do} + C_{dcl}(C_{lo}$

$+ C_{la}x_1 + C_{lf}u_2)^2 + C_{df}u_2 + N\epsilon(C_{to}$

$+ C_{tv}x_2 + C_{th}x_5 + C_{tvh}x_2 x_5)(C_{bo} + C_{bn}N$

$+ C_{bh}x_5 + C_{bp}(C_{po} + C_{pn}N + C_{ph}x_5 + C_{pnt}Nu_3))$

2.8) $\quad \dot{x}_3 = x_4$

2.9) $\quad \dot{x}_4 = \frac{1}{I_{yy}}(\frac{1}{2}\rho x_2^2 Sc)[C_{mo} + C_{mcl}(C_{lo}$

$+ C_{la}x_1 + C_{lf}u_2) + C_{mf}u_2 - \frac{c}{2x_2}(\dot{x}_1 + x_4)]$

2.10) $\quad \dot{x}_5 = x_2\sin(x_3 - x_1)$

Table 4

States and controls

$x_1 = \alpha$	$u_1 = \delta_e$
$x_2 = V$	$u_2 = \delta_f$
$x_3 = \theta$	$u_3 = \delta_t$
$x_4 = \dot{\theta}$	
$x_5 = H$	

2.2 LINEARIZATION

In order to use linear control theory, it is necessary to obtain a linearized version of the system. This is accomplished using first order perturbation techniques.

$$\dot{x} = f(x,u,t)$$

$$\dot{x} = \frac{\partial f}{\partial x} x + \frac{\partial f}{\partial u} u$$

Define:

$$A = \frac{\partial f}{\partial x}, \quad B = \frac{\partial f}{\partial u}$$

2.3 EQUILIBRIUM

To proceed we must now find an equilibrium point. Given a desired set point, subject to the restrictions below, one can find what the control values should be at equilibrium.

$$\dot{x}_e = 0 = f(X,U)$$

where X and U are equilibrium values.

Given the desired equilibrium state X,

since $\dot{x}_3 = 0$ and $\dot{x}_5 = 0$ we get,

2.11) $X_4 = 0$

2.12) $X_3 = X_1$

From $\dot{x}_1 = 0$ we see that:

2.13) $U_2 = \dfrac{1}{C_{1f}} (-C_{1o} - C_{1a}x_1 + \dfrac{W}{QS}\cos(X_3-X_1) + MX_4X_2)$

$$= \dfrac{1}{C_{1f}}(-C_{1o} - C_{1a}X_1 + \dfrac{W}{QS})$$

2.14) $U_1 = \dfrac{-1}{C_{me}}(C_{mc1}C_1 + C_{mo} + C_{mf}U_2)$

2.15) $U_3 = \dfrac{-1}{C_{bp}C_{pnt}N}(C_{bo} + C_{bp}C_{po} + N(C_{bn} + C_{bp}C_{pn})$

$$+ (C_{bh} + C_{bp}C_{ph})X_5 - \dfrac{QS}{Tbhp}C_d$$

where,

$$Q = \tfrac{1}{2}\rho X_2^2$$

$$C_1 = C_{1o} + C_{1a}X_1 + C_{1f}U_2$$

$$C_d = C_{do} + C_{dc1}C_1^2 + C_{df}U_2$$

$$Tbhp = Ne(C_{to} + C_{tv}X_2 + C_{th}X_5 + C_{tvh}X_2X_5)$$

Plugging in these equilibrium values and linearizing yields the coefficients of A and B given in Tables 5 and 6. Now our linearized plant with feedback can be modeled as in the block diagram given in Fig.2.

Table 5

Linearized 'A' Coefficients

$$a_{11} = -\frac{QS}{MX_2} C_{la}$$

$$a_{12} = -\frac{\rho S}{2M} C_1 - \frac{W}{MX_2^2}$$

$$a_{14} = 1.$$

$$a_{21} = g - 2Q\frac{S}{M} C_{dcl} C_{la} C_1$$

$$a_{22} = -X_2 C_d \frac{S\rho}{M} + Ne(C_{tv} + C_{tvh}X_5)\frac{Bhp}{M}$$

$$a_{23} = -g$$

$$a_{25} = \frac{Ne}{M}[(C_{th} + C_{tvh}X_2)Bhp + Tbhp(C_{bh} + C_{bp}C_{ph})]$$

$$a_{41} = \frac{QSc}{I_{yy}} (C_{mcl}C_{la} - \frac{c}{2X_2} a_{11})$$

$$a_{42} = \frac{\rho X_2 Sc}{I_{yy}}[(C_{mo} + C_{mcl}C_{lo}) + C_{mcl}C_{la}X_1 + C_{me}U_1$$

$$+ (C_{mcl}C_{lf} + C_{mf})U_2$$

$$+ \frac{Sc^2\rho}{4I_{yy}}(-2X_4 + \frac{\rho}{M}X_2 SC_1)$$

$$a_{43} = -\frac{QSc^2}{2I_{yy}X_2} a_{13}$$

$$a_{44} = -\frac{QSc^2}{I_{yy}X_2}$$

$$a_{51} = -X_2$$

$$a_{53} = X_2$$

$$a_{13} = a_{15} = a_{24} = a_{31} = a_{32} = a_{33} = a_{35} = a_{45}$$

$$= a_{52} = a_{54} = a_{55} = 0$$

Table 6

Linearized 'B' coefficients

$$b_{12} = -\frac{QS}{MX_2} C_{1f}$$

$$b_{22} = -S\frac{Q}{M}(2C_{dc1}C_{1f}C_1 + C_{df})$$

$$b_{23} = T_{bhp}C_{bp}C_{pnt}N$$

$$b_{41} = \frac{QSc}{I_{yy}}C_{me}$$

$$b_{42} = \frac{QSc}{I_{yy}}(C_{mc1}C_{1f} + C_{mf} - \frac{c}{2X_2}b_{12})$$

$$b_{11} = b_{13} = b_{21} = b_{31} = b_{32} = b_{33}$$
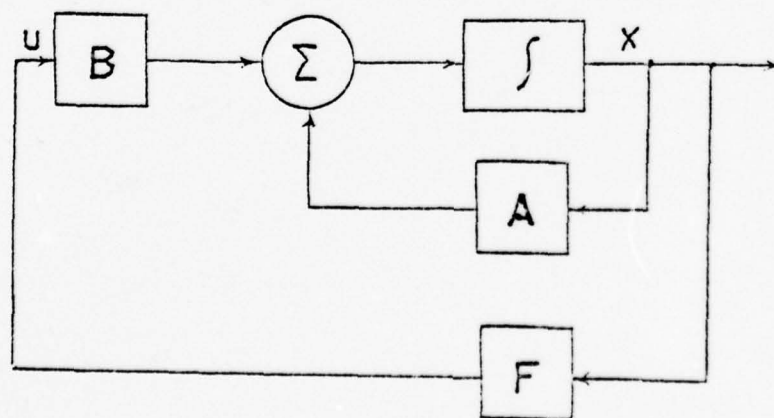
$$= b_{43} = b_{51} = b_{52} = b_{53} = 0$$

_____

Figure 2. Linearized Plant Block Diagram

CHAPTER 3

CONTROLLER DESIGN

3.1 FEEDBACK CONTROL

Since a pilot is generally interested in obtaining a desired velocity, altitude, or pitch angle it would be favorable to have controls which directly relate to these quantities. As it is, he must estimate how much more throttle he must use to get the airplane to a certain velocity, and similar situations exist for altitude control. This is not a critical problem, usually, since there is little need for perfect accuracy when one is 20,000 ft. in the air. But in order to conserve fuel or more importantly, when landing in a heavy storm this can become critical. One other application is for pilotless control of an aircraft, as is the case for missiles and drones.

Assuming that the state variable vector, X, is available (or at least estimated) a logical choice for a control is one which the control variables are a linear combination of the state variables. In other words the control can be written:

$$u = Fx$$

Now in order to analyze the stability of the system with this control, the following linearized system is examined:

$$\dot{x}=Ax + Bu =Ax +BFx=(A + BF)x$$

3.1.1 Pole Placement Technique

Using pole placement techniques, the eigenvalues of this system can be placed arbitrarily if the system is controllable (Ref.1).In other words if the matrix $(B,AB,A^2B,...,A^{n-1}B)$ has rank n, a desired response can be obtained. Both the controllability test and the placement of eigenvalues can be accomplished using Linsys (Ref.2). The poles are placed in the left half plane so that as t gets large $\dot{x}=0$. This characteristic will be utilized later.

3.1.2 Optimal Control - the Minimum Principle

An alternate method of finding the control feedback matrix, is to assume a performance index J, that we want to minimize. If we assume our performance index to be of the form:

$$J=\int_0^\infty (x^TQx + u^TRu) dt$$

where Q and R are cost matrices which can be adjusted to procure a certain response. If A, B, and Q conform to the conditions stated in Appendix B then there exists a K, such that the system with the following control is asymptotically

stable (Ref. 1), where K is found from the algebraic Riccati equation:

$$-KA - A^T K + KBR^{-1}B^T K - Q = 0$$

$$\text{and } u = Fx = -R^{-1}B^T Kx$$

The above Riccati equation and the resulting F can all be found using Linsys (Ref. 2).

3.1.3 Augmentation

Since both of these above methods result in systems which are asymptotically stable, we can use this fact to 'force' a state or states to a specified set point value. We note that:

$$\dot{x}(\infty) = 0$$

$$y = Cx$$

If we now want $y(\infty) = y_{ref} = Cx$, we can augment our original system and define $\dot{x}'$ such that:

$$\dot{x}' = \begin{bmatrix} \dot{x} \\ y - y_{ref} \end{bmatrix} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} x' + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ -y_{ref} \end{bmatrix}$$

or

$$\dot{x}' = A'x' + B'u + v$$

and with proper feedback:

$$\dot{x}'(\infty) = 0 \quad \text{and} \quad y(\infty) = y_{ref}$$

In our case we want to control velocity, V, and pitch angle, θ , so:

$$\dot{x}_6 = x_2 - V_{ref}$$

$$\dot{x}_7 = x_3 - \theta_{ref}$$

and $\quad C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$

3.2 OBSERVER DESIGN

Throughout the rest of this thesis we will assume that certain states are not actually available for measurement (or perhaps not very accurate). Since for accurate and efficient controlling (or perhaps for pilots reference) these states are needed, some technique is necessary to provide these. For this an 'observer' or 'state estimator' is used which, as the names imply, estimate the state or states that can't be directly measured . For the purposes of this thesis (and to demonstrate this technique) it will be assumed that the states that are not available are angle of attack and pitch rate. In reality these may be measurable, but for reasons such as cost or precision it will be assumed that they are not accurately available. On the other hand the 3 states - pitch angle, velocity, and altitude have to be measured for the pilots referance, anyway, so these certainly would be available, and would not require any further instrumentation.

There may be a realistic limit on the order of the observer, due to computer size, speed or cost, so a low order observer will be used. Using the results obtained in Appendix A:

$$A = TAT^{-1} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$$B = TB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

$$\dot{w}_2 = G_1 w_2 + G_2 y + G_3 u$$

where:

$$G_1 = A_{22} - KA_{12}$$

$$G_2 = A_{21} - KA_{11} + G_1 K$$

$$G_3 = B_2 - KB_1$$

$$w_2 = \hat{z}_2 - Kz_1$$

Because of the special form of the transformation matrix, T, the actual implementation is made somewhat simpler. This is partly due to the fact that 3 of the states are outputs, and part due to the choice of M. The matrix, T, is chosen as:

$$T = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

As is easily seen x is obtained simply by interchanging the order of z_i. So the estimated states $\hat{z}_2$ are actually x_1 and x_2, respectively. It might also be noted here that it is assumed that x_6 and x_7 are available since they are needed previously for the augmentation. Taking this into account results in the block diagram of Fig.3. The nonlinear version is shown in Fig.4, where the observer-controller is the same as in the linear case, except that the controls and states are normalized around

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Figure 3. Linear Observer–Controller Block Diagram

the equilibrium point. A slightly different approach can also be taken in arriving at an observer-controller design. This Smith-Davison method (Ref. 5,8) is a more detailed approach for disturbance rejection, but yields the same results obtained here.
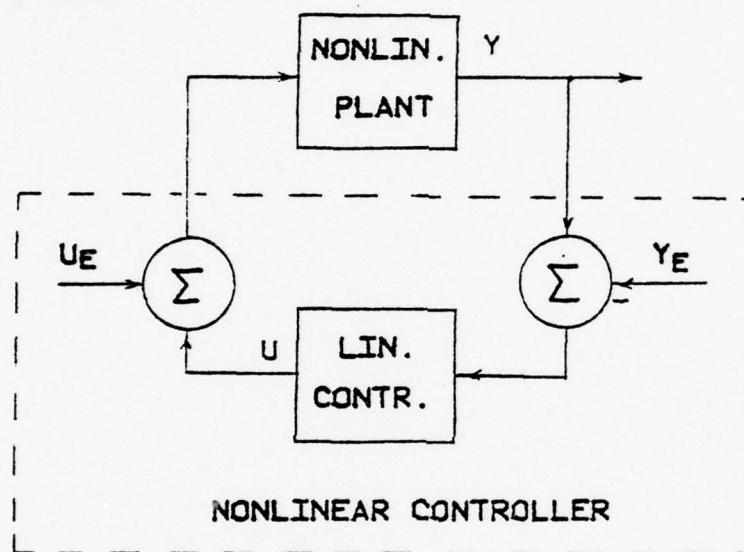

Figure 4. Nonlinear Observer-Controller Block Diagram

3.3 CONTROLLER RESULTS

3.3.1 Feedback Matrix

The cost matrices Q and R were chosen by adjusting the corresponding elements to yield a desired response. A desired response was considered to satisfy the following criteria:

1. Fairly quick settling time to the set point (5 sec.).

2.   Relatively smooth response, with few, if any, jumps or jerks.

3.   As little overshoot as possible.

4.   Keeping controls within bounds and limitations.

Following this set of conditions the cost matrices were arrived at. Upon substitution into the Riccati package of Linsys the feedback matrix, F, was calculated. The eigenvalues for this linear system, along with the rest of the system matrices, are shown in Appendix D. It should be noted here that in order to use the Riccati package the matrices were normalized so that all elements were about of the order of one (x_2, x_5, x_6 are scaled down by 100).

The observer feedback matrix was similarly chosen using the Riccati package. The values obtained for K and the corresponding Q, R, and eigenvalues are given in Appendix D.

3.3.2 Effects of Parameter Variations

In order to test how effective the controls are in simulated flight conditions, several types of parameter variations were used. All of the variations are realistic disturbances that an aircraft can encounter. The time responses due to these variations appear in Figs. 5.1-5.7 on pages 24-34.

3.3.2a Perturbations

The first type of disturbance studied was simply a change in initial condition, which could represent a gust of wind, for instance. As one can see from the plots of Figs. 5.1a and 5.2a, the controller (with observer) performed properly as long as these perturbations were within reason(i.e. within 30 ft/sec or .2 rad).

3.3.2b Disturbance

For this a constant disturbance vector is introduced into the plant's differential equations, this could correspond to a headwind or perhaps inaccurate readings. Figs. 5.3a and 5.3c show the responses with this disturbance introduced. For a small disturbance the response was hardly affected. As the disturbance was increased the response showed signs of instability and the response became oscillatory.

3.3.2c Set Point

The set point is the desired flight condition as specified by the pilot (e.g. velocity or pitch angle). An example of this is shown in Figs. 5.4a and 5.4c. For set points far from the original conditions the response became unfavorable and often the controls tended to exceed their realistic limits.

3.3.2d Plant Variations

It was also possible to change the different aircraft parameters, such as center of gravity, to represent a shift of cargo as in Figs. 5.6a and 5.6b, for instance. Bringing the center of gravity farther forward had little effect until around .5 ft., where depending on whether an observer was used or not the system went unstable, with the observer system going unstable earlier. Bringing the center of gravity farther back in the plane tended to yield better responses than at the nominal condition, and evidently created a more stable situation. This agrees with what is expected from aircraft dynamics, the farther back the center of gravity the more stable it should be by design.

3.3.3 Effects of Using an Observer

The use of the observer did not seem to drastically affect the response of the system. As can be seen from the plots of Fig.5.1 there is no significant difference after the first 2 seconds. The observer system did seem to go unstable sooner, as variations approached their reasonable limits. But the observer also tended to 'smooth' the response somewhat, which is a desirable side effect of its use.

3.3.4 Real Time Results

To simulate actual aircraft implementation, the controller designed in this chapter was tested on the AD5-PDP11 hybrid computer. The continuous model was used here rather than convert to a discrete model. This is seen to be a safe simplification for high enough sampling rates. Examples of the responses are shown in Figs. 5.1, 5.3, and 5.4 (b and d).

3.3.4a Sampling Rate Effects

The system seemed to be very sensitive to the sampling rate. It was found that the digital controlling program took around .01 seconds to run. But the system with controller tended to go unstable with .08 second sample time. This is a rather limited range of sampling rates and would tend to be a drawback for real time operation.

3.3.4b Observer Effects

Here, as was the case for the continuous time simulation, the observer use had little effect on the response. It tended to also smooth the response somewhat which helped to keep the controls in bounds. It was also noted that the observed states had little effect in the first place. This was discovered by not feeding these states back at all.
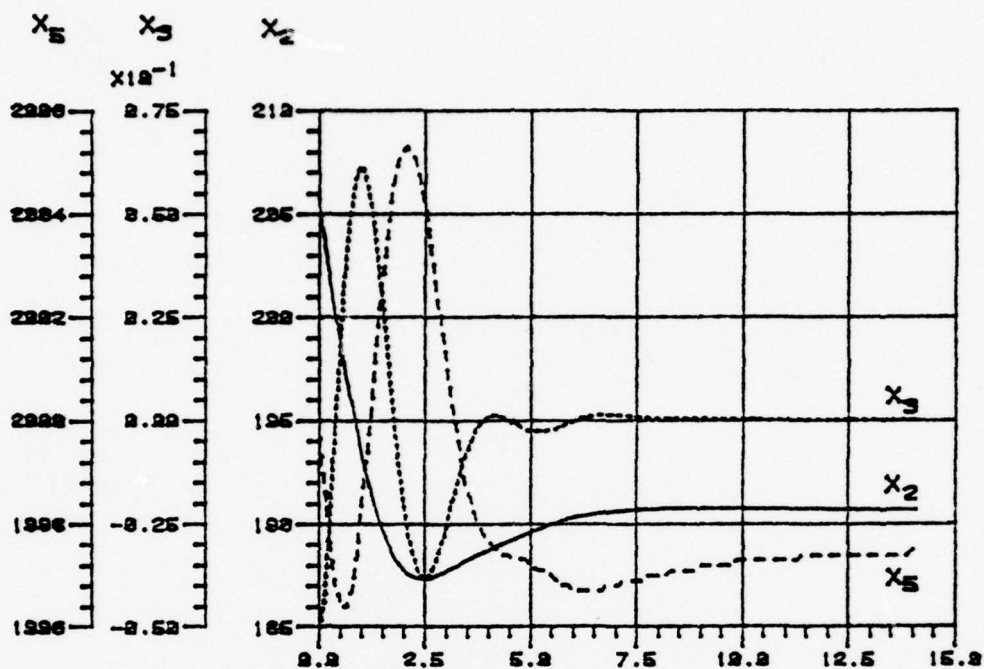
Figure 5.1a   State Response with Observer
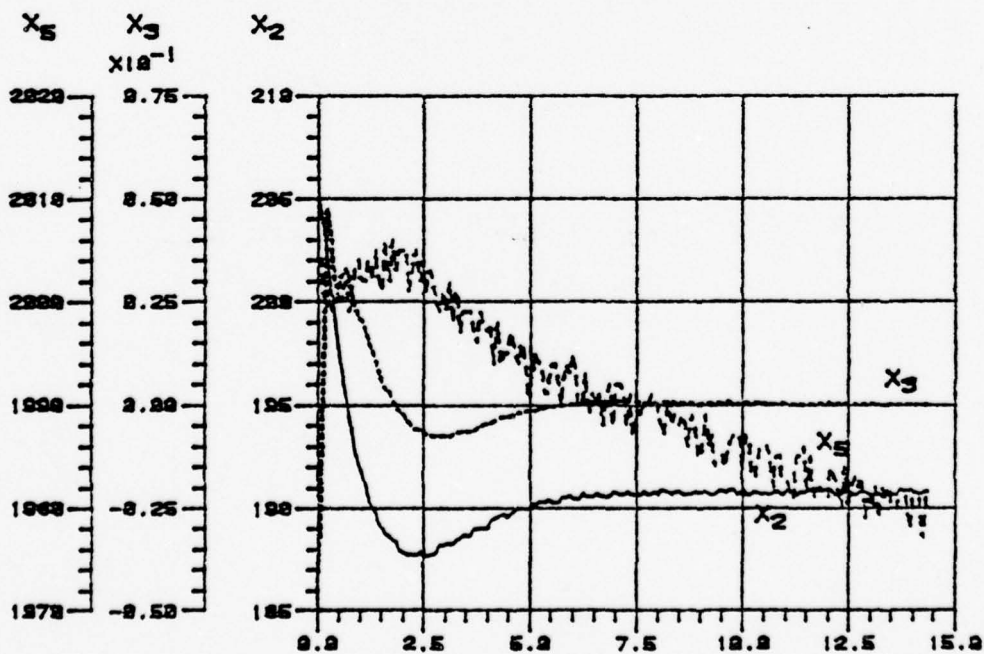
$X_{2per}= 15.$ $X_{3per}= -0.05$

Figure 5.1b   State Response of Hybrid System
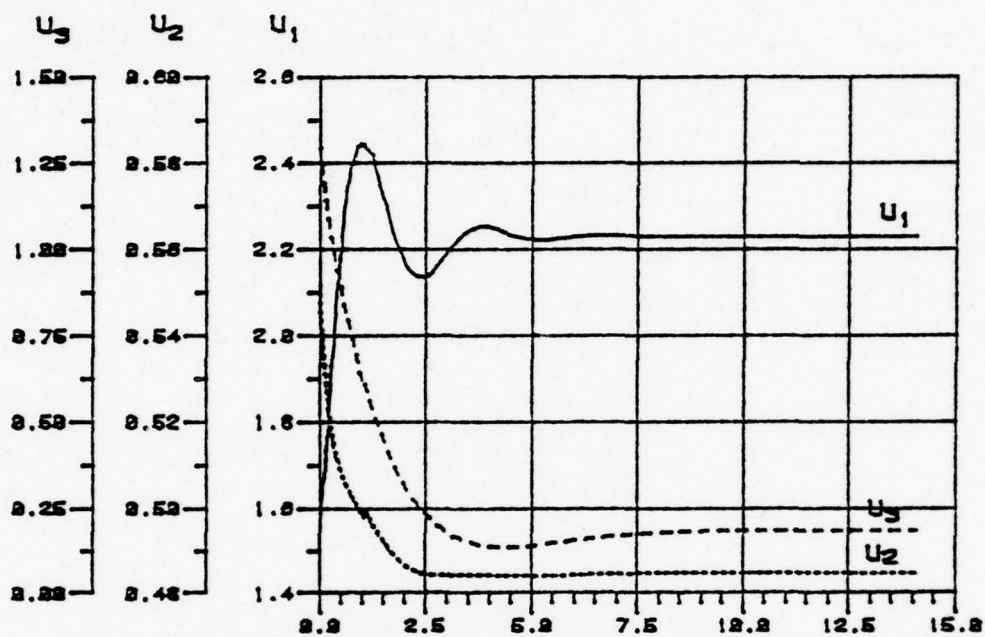
$X_{2per}= 15.$ $X_{3per}= -0.05$

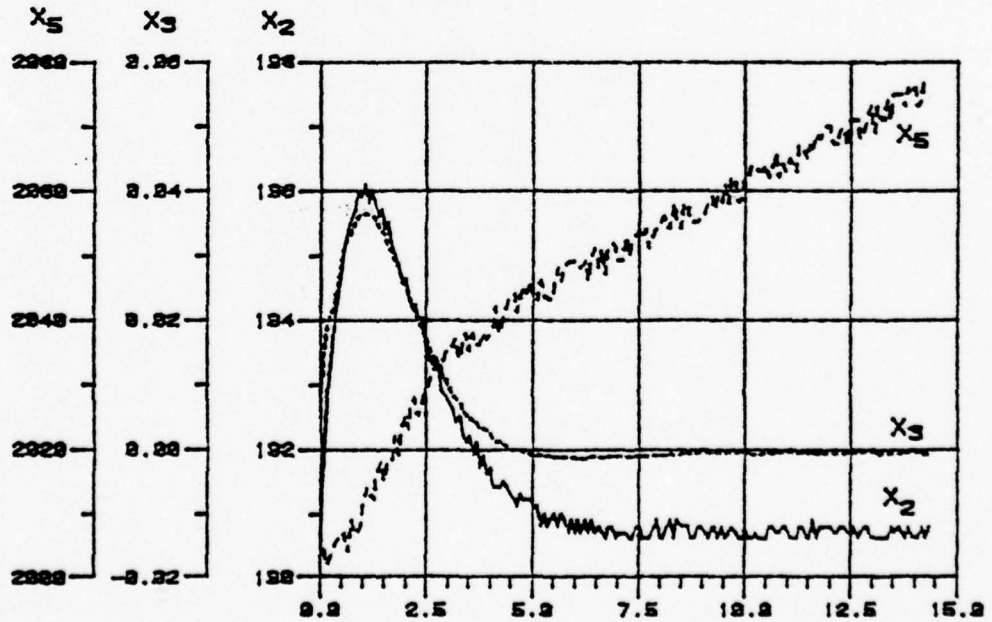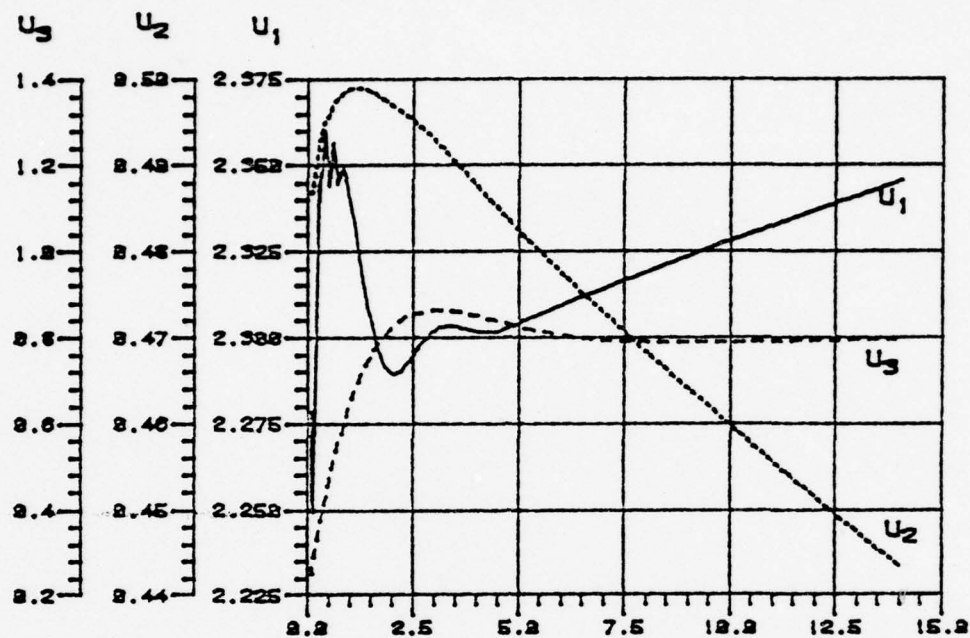Figure 5.1c   Control Response with Observer
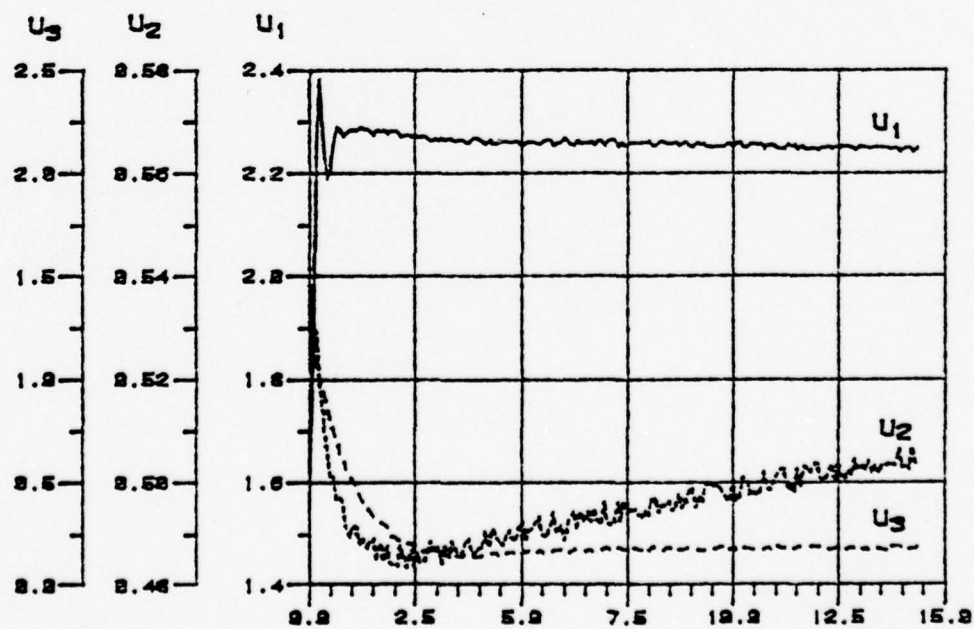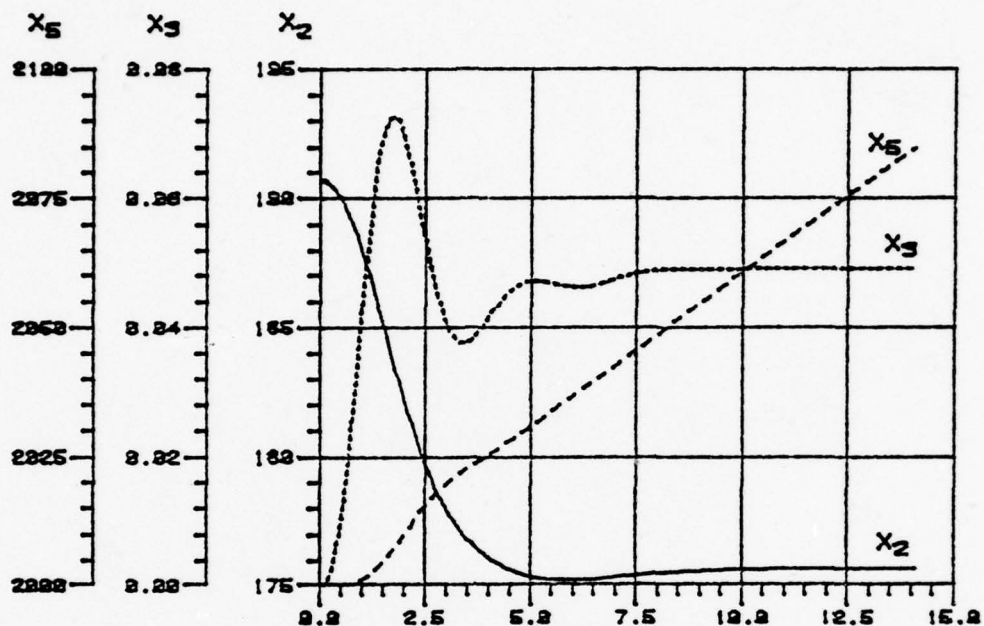
$X_{2per} = 15.$ $X_{3per} = -0.05$

Figure 5.1d   Control Response of Hybrid System
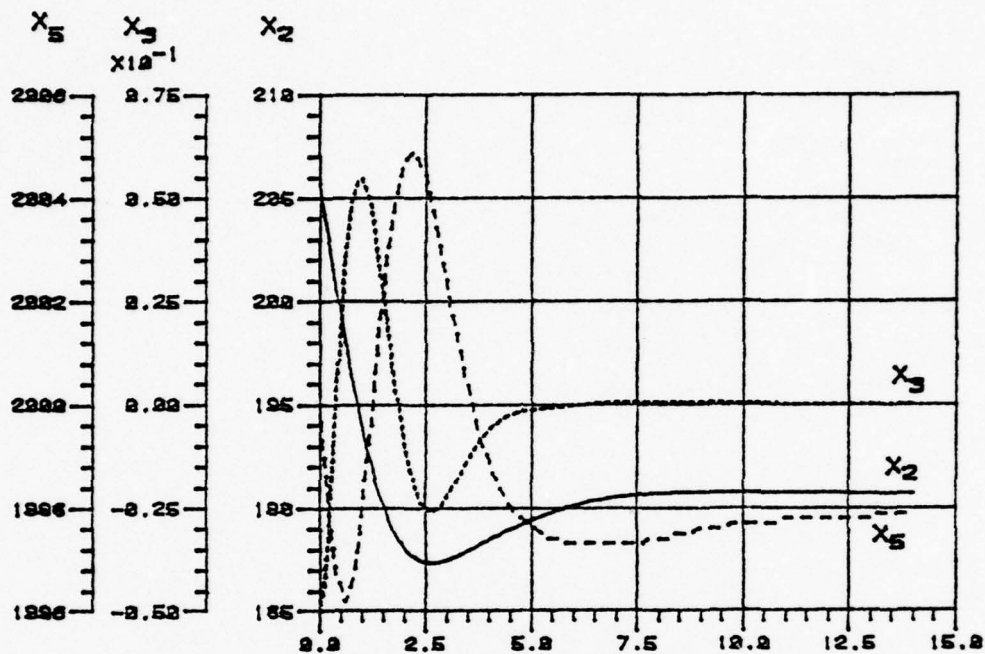
$X_{2per} = 15.$ $X_{3per} = -0.05$

Figure 5.1e State Response with all States Available
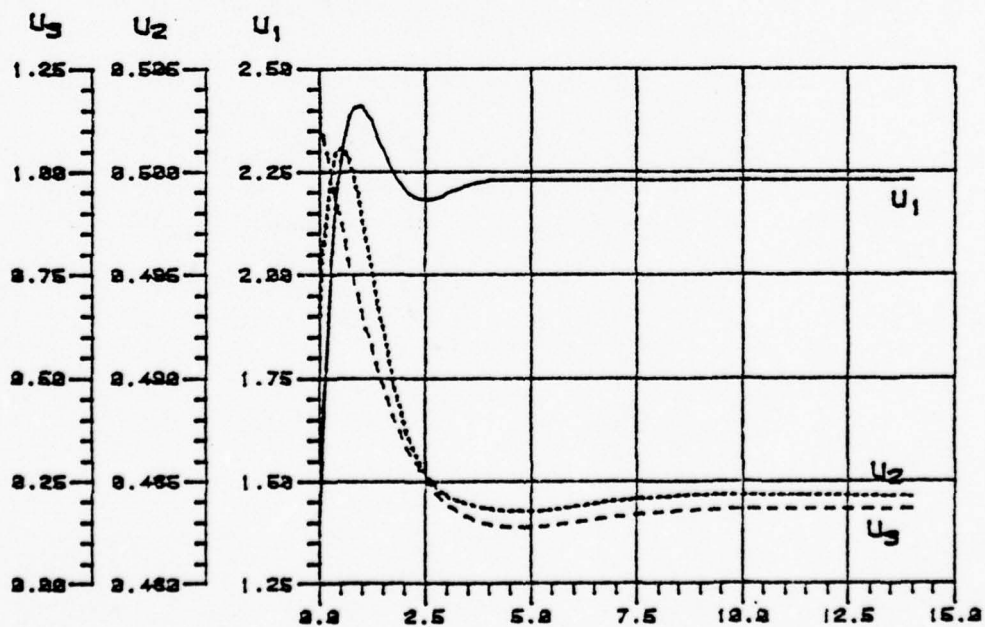
X_{2per} = 15. X_{3per} = -0.05

Figure 5.1f Control Response with all States Available

X_{2per} = 15. X_{3per} = -0.05
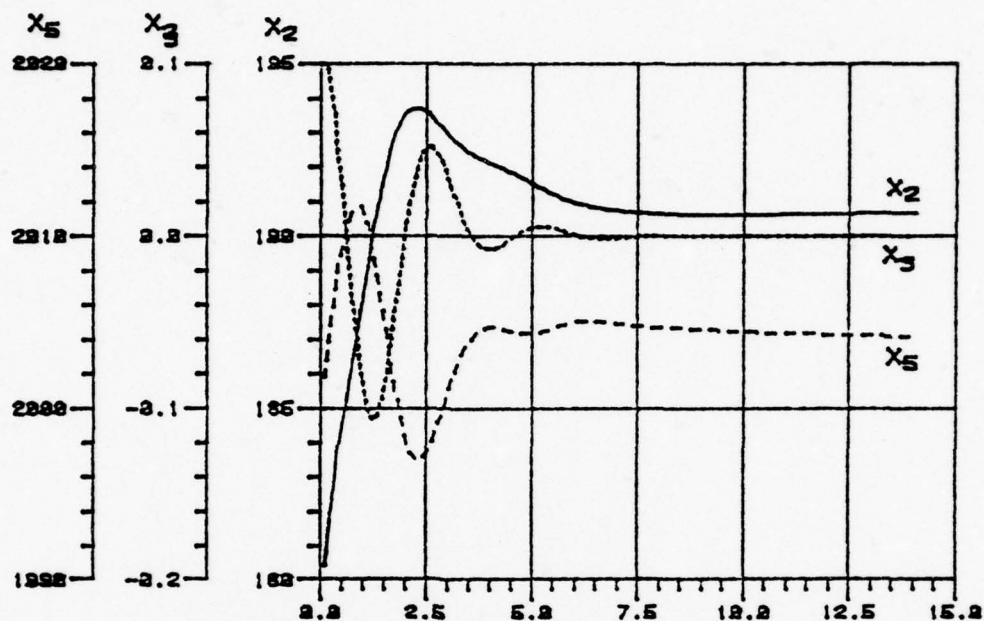
Figure 5.2a   State Response with Observer
X_{2per}= -10. X_{3per}= 0.1
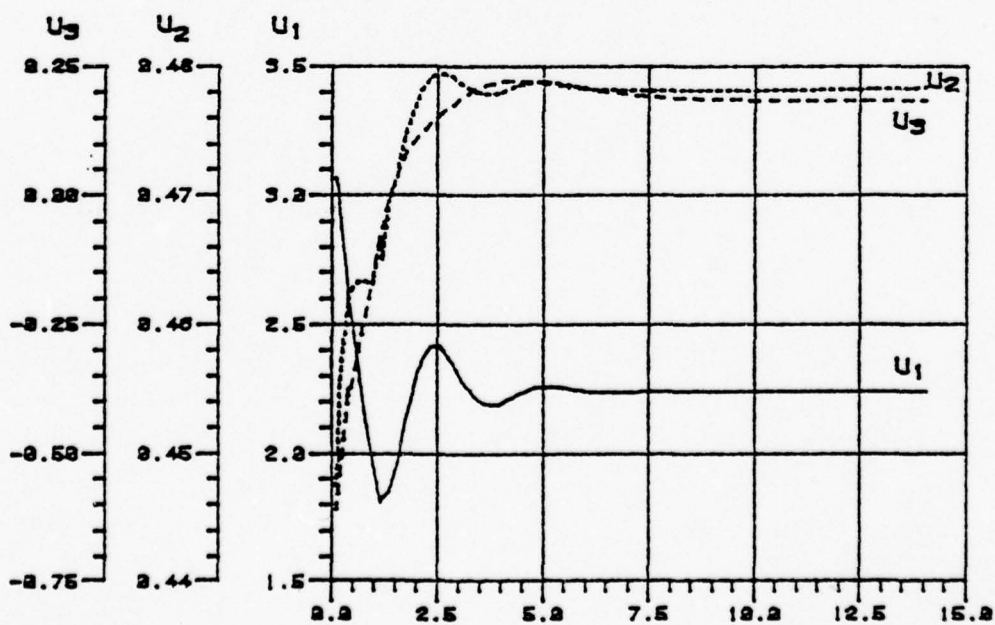
Figure 5.2b   Control Response with Observer
X_{2per}= -10. X_{3per}= 0.1
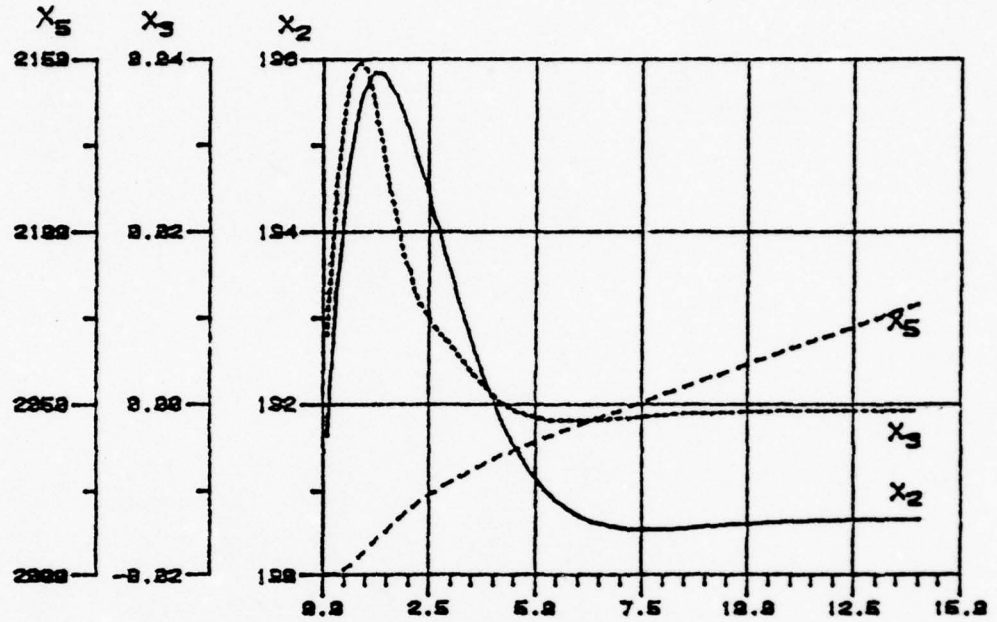
Figure 5.3a   State Response with Observer
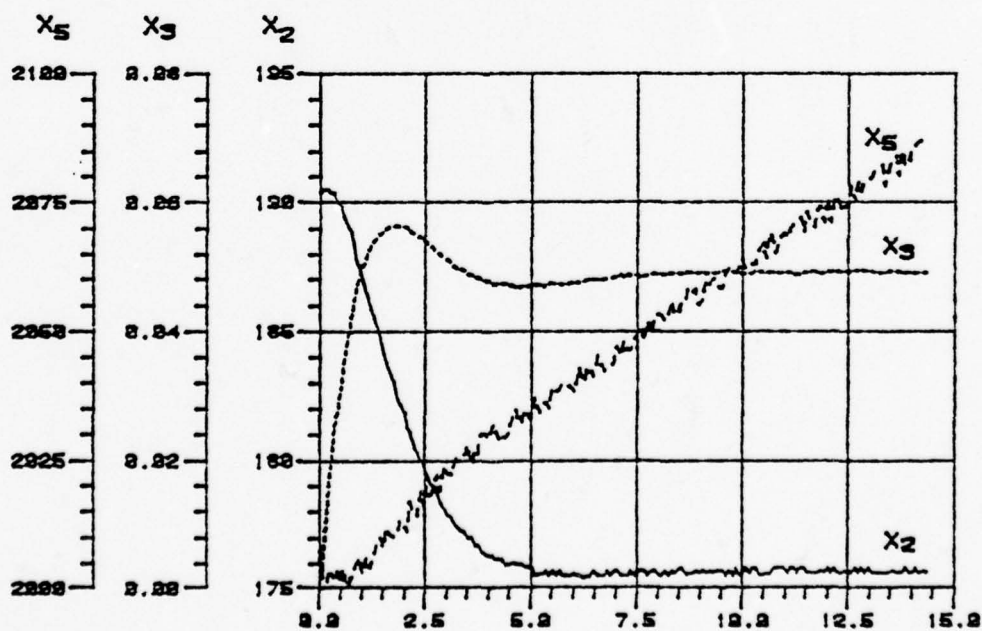
X_{2dis}= 10. X_{3dis}= 0.08

Figure 5.3b   State Response of Hybrid System

X_{2dis}= 10. X_{3dis}= 0.08

Figure 5.3c   Control Response with Observer
X_{2dis}= 10. X_{3dis}=0.08
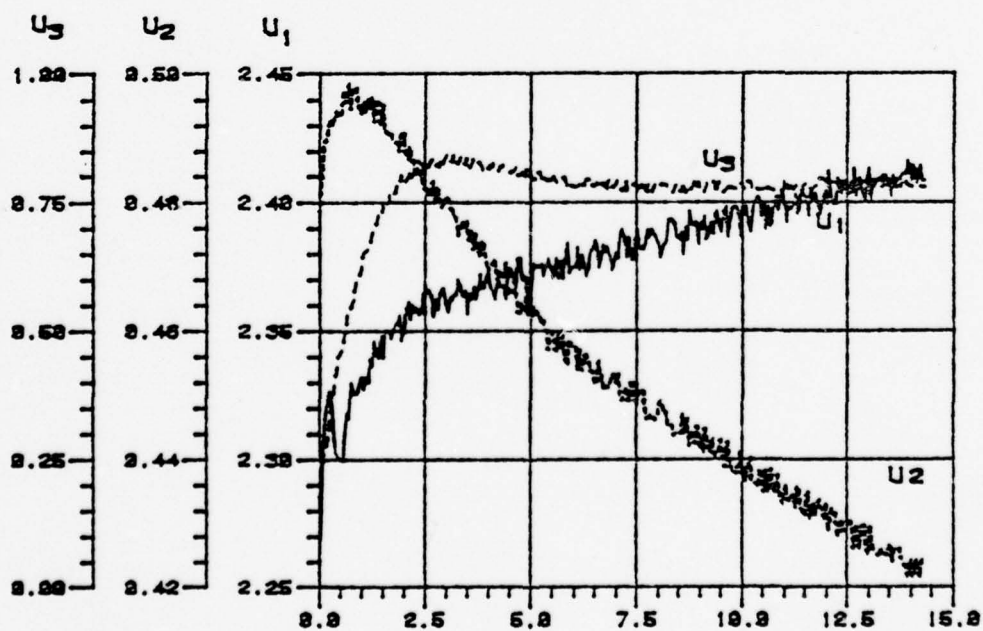
Figure 5.3d   Control Response of Hybrid System
X_{2dis}= 10. X_{3dis}= 0.08

Figure 5.4a   State Response with Observer
$X_{2set} = -15.$ $X_{3set} = 0.05$

Figure 5.4b   State Response of Hybrid System
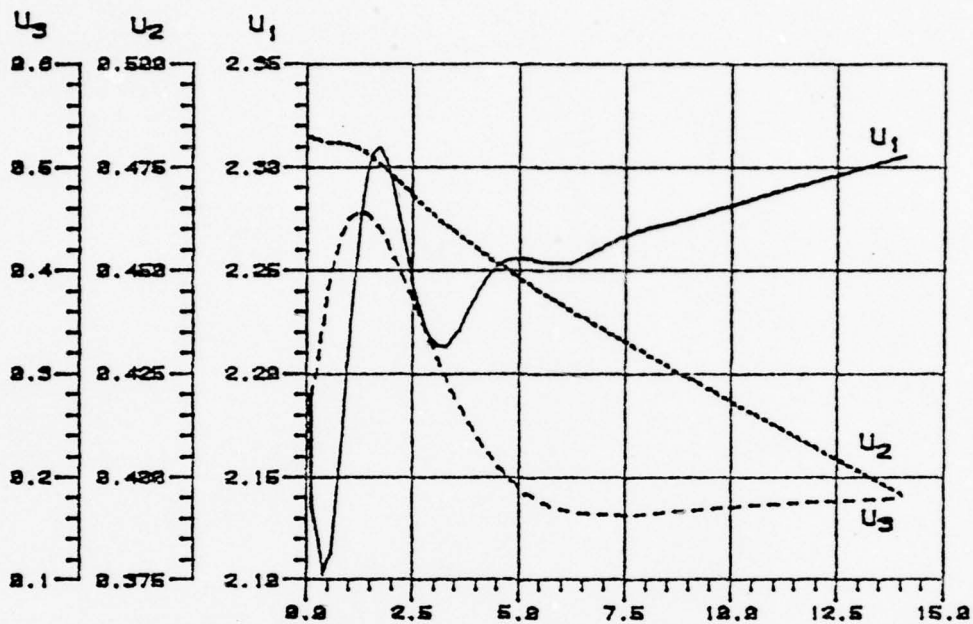$X_{2set} = -15.$ $X_{3set} = 0.05$

Figure 5.4c Control Response with Observer
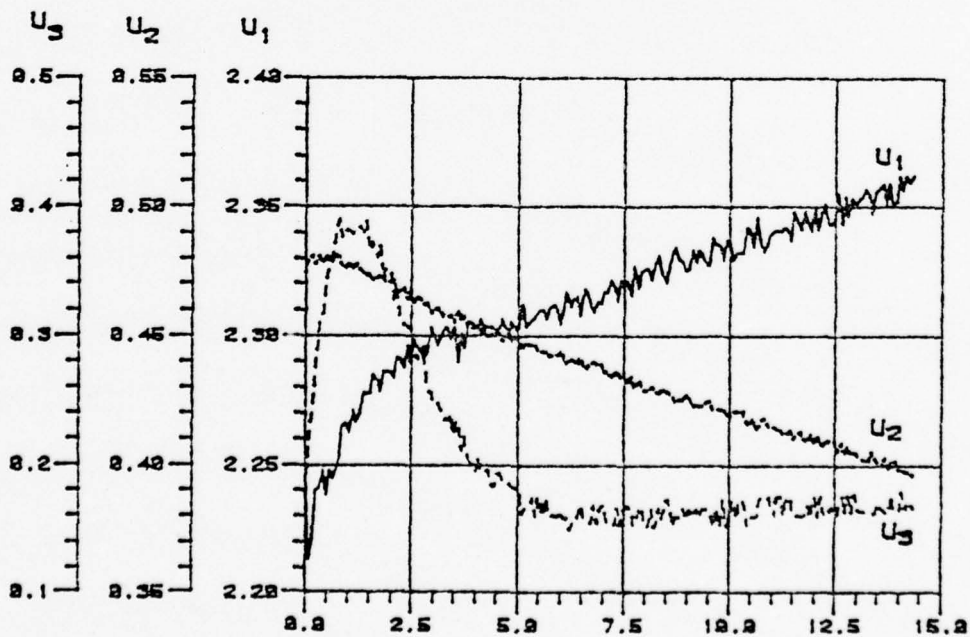$X_{2set} = -15.$ $X_{3set} = 0.05$

Figure 5.4d Control Response of Hybrid System
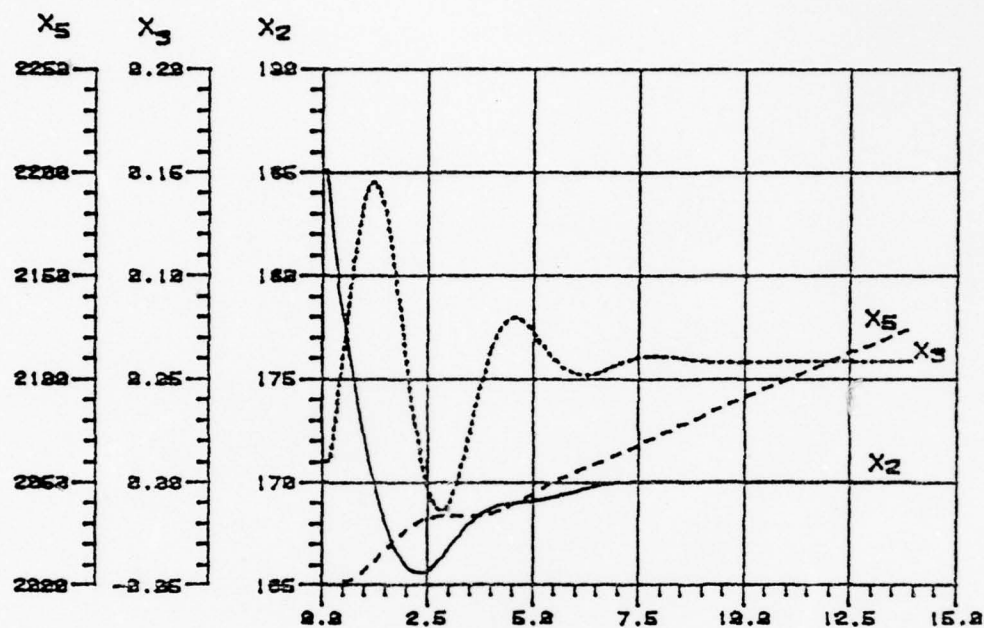$X_{2set} = -15.$ $X_{3set} = 0.05$

Figure 5.5a State Response with Observer, Equilibrium Shift

X_{2per} = 15. X_{3per} = -0.05 X_{2e} = 170. X_{3e} = 0.06
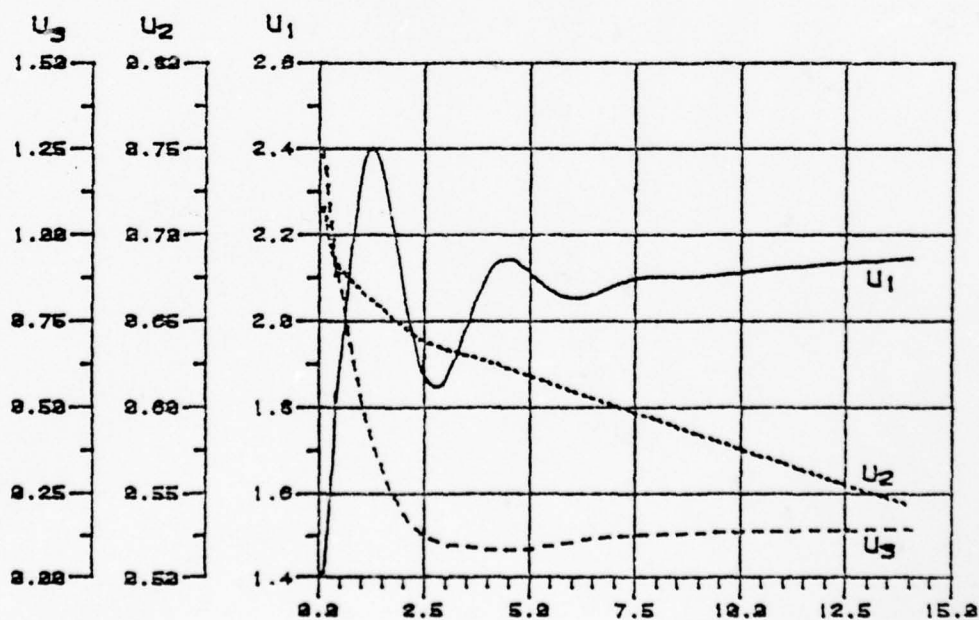

Figure 5.5b Control Response with Observer, Equilibrium Shift

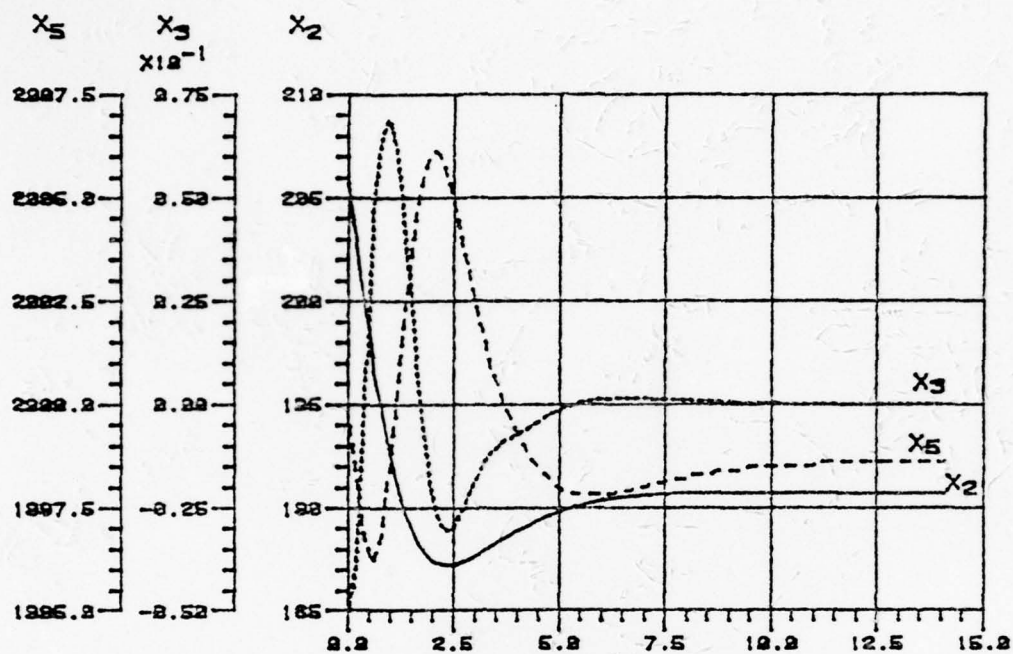X_{2per} = 15. X_{3per} = -0.05 X_{2e} = 170. X_{3e} = 0.06

Figure 5.6a State Response with Observer, X_{cg} shifted -25%

X_{2per}= 15. X_{3per}= -0.05
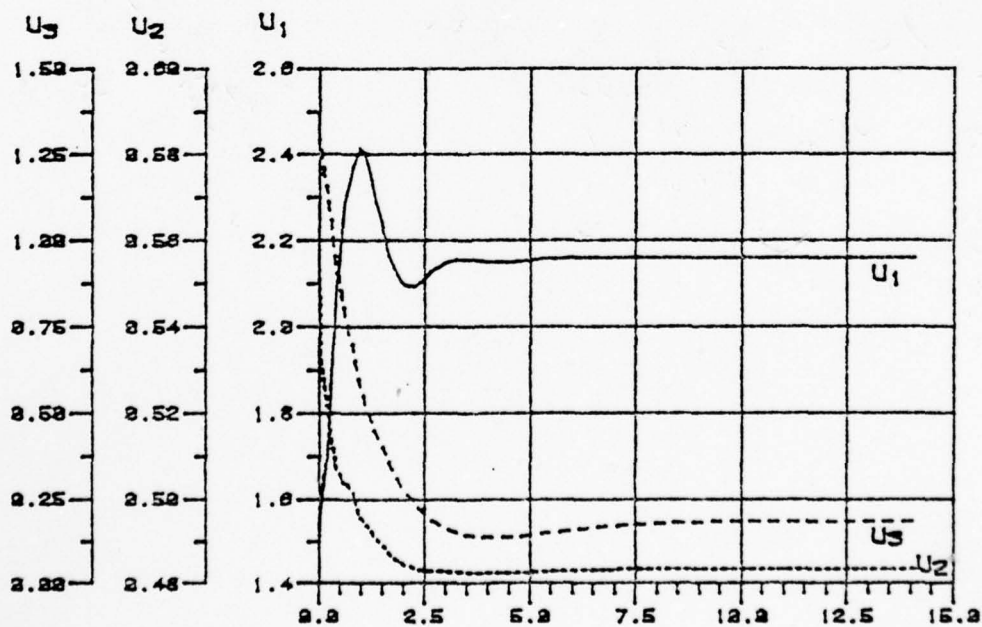

Figure 5.6b Control Response with Observer, X_{cg} Shifted -25%

X_{2per}= 15. X_{3per}= -0.05

Figure 5.7a   State Response with Observer, ρ Shifted -25%

$X_{2per} = 15.$ $X_{3per} = -0.05$

Figure 5.7b   Control Response with Observer, ρ Shifted -25%
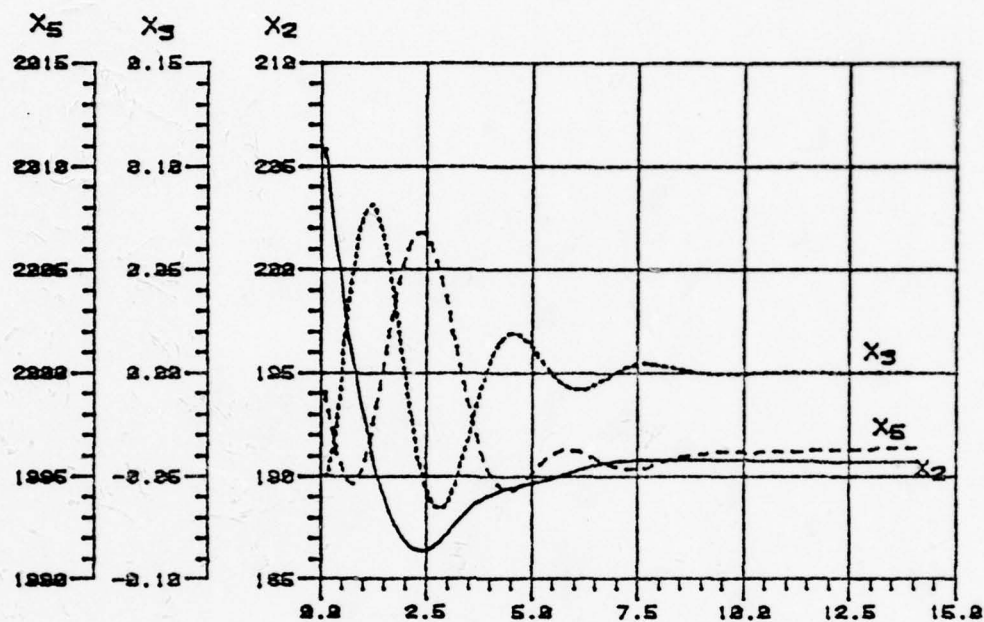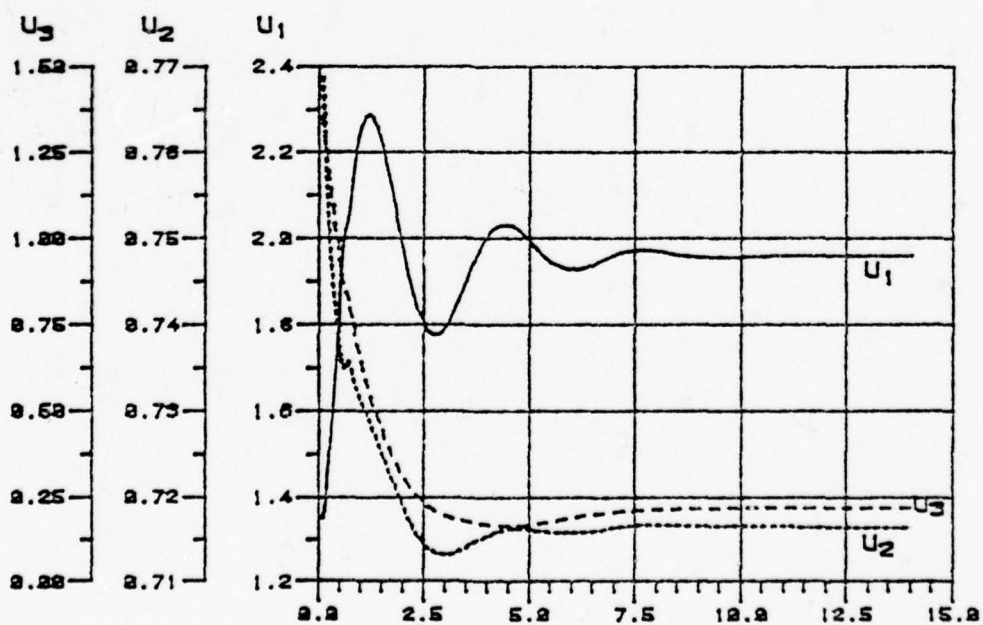
$X_{2per} = 15.$ $X_{3per} = -0.05$

3.3.4c Real Time Effects

The responses were somewhat more jumpy probably because of the analog to digital conversion which would reduce the accuracy, especially in the case of altitude. In general, though, the responses in real time were nearly identical to the continuous time counterpart, and if anything tended to have less overshoot.

3.3.5 Operating Envelope

When conditions were varied too far from the nominal flight conditions the response characteristics deteriorated. With this in mind, different pertinent parameters were varied to determine a region for which this controller would still operated satisfactorily. This operating envelope is found to be approximated by the following boundaries with the corresponding characteristics making the response undesirable.

1. Velocity $170 < V < 240$. At 170 ft/sec the pitch angle is oscillatory. At 240 ft/sec the throttle control goes out of bounds.

2. Pitch Angle $-0.25 < \alpha < 0.1$. The throttle goes out of bounds at both of these limits.

3. Disturbance, velocity: 10 ft/sec^2 pitch angle: 0.2

rad/sec. The throttle goes out of bounds for both of these.

4. Center of Gravity $-3.0 < X_{cg} < .25$. At $X_{cg} = 0.25$ the system becomes oscillatory. At $X_{cg} = -3.0$ the response is sluggish and the controls go out of bounds.

5. Air Density $0.003 < \rho < 0.01$. At the lower bound the system oscillates while at the upper limit the throttle goes out of bounds.

CHAPTER 4

SENSITIVITY

Sensitivity is the analysis of how a parameter variation affects the dynamics of a system. For instance, by shifting an aircraft's center of gravity (e.g. fuel or cargo moving) too far forward, an inherently stable aircraft can become unstable, making it harder to control. So one purpose of feedback control is to hopefully reduce or eliminate any tendency toward instability. In this chapter, the sensitivity of the controlled system and the observer controlled system are compared.

4.1 SENSITIVITY MODEL

By definition, trajectory sensitivity is the change in trajectory due to a change in the parameter p.

$$\lim_{\Delta p \to 0} \frac{\Delta x}{\Delta p} = \frac{\partial x}{\partial p}$$

Or normalizing this with respect to p:

$$\lim_{\Delta p \to 0} \frac{\Delta x}{\frac{\Delta p}{p}} = p \frac{\partial x}{\partial p}$$

If we are given a system of the form:

$$\dot{x} = Ax + Bu$$

taking the partial derivative of this equation with respect to p yields:

$$\frac{\partial \dot{x}}{\partial p} = A \frac{\partial x}{\partial p} + B \frac{\partial u}{\partial p} + \frac{\partial A}{\partial p}x + \frac{\partial B}{\partial p}u$$

Here we assume that u does not depend on p. If u utilizes feedback control, u = Fx, then we assume A' = (A + BF), where F does not depend on p. Letting

4.1) $\quad \frac{\partial \dot{x}}{\partial p} = A' \frac{\partial x}{\partial p} + \frac{\partial A}{\partial p}x + \frac{\partial B}{\partial p}u = A' \frac{\partial x}{\partial p} + (\frac{\partial A}{\partial p} + \frac{\partial B}{\partial p}F)x$

Now we have an nth order sensitivity index, p , which requires x and u as its inputs or for simple feedback control just requires x. For this system, the block diagram is as given in Fig.6.
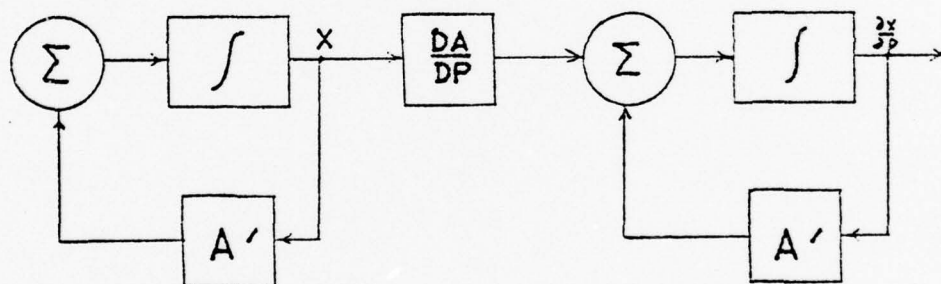

Figure 6. System Sensitivity Block Diagram

If an observer is used this complicates the situation considerably, because instead of direct feedback control we use the observed value's feedback:

u = Fx = FTz

We assume that F and T do not depend directly on p, so:

$$\frac{\partial u}{\partial p} = \frac{\partial F}{\partial p}Tz + F\frac{\partial T}{\partial p}z + FT\frac{\partial z}{\partial p} = FT\frac{\partial z}{\partial p}$$

The job now is to find how z depends on p. From the observer chapter we know:

$$\hat{z} = \begin{bmatrix} z_1 \\ \hat{z}_2 \end{bmatrix}$$

$$\hat{z}_2 = w_2 + Kz_1$$

As before $\frac{\partial z}{\partial p}1$ is available simply in the form of $\frac{\partial y}{\partial p}$.

$$\frac{\partial \hat{z}}{\partial p}2 = \frac{\partial w}{\partial p}2 + K\frac{\partial y}{\partial p}$$

So $\frac{\partial w}{\partial p}2$ is now needed.

$$\dot{w}_2 = G_1 w_2 + G_2 y + G_3 u$$

So taking the partial derivative with respect to p:

4.2) $$\frac{\partial \dot{w}}{\partial p}2 = G_1\frac{\partial w}{\partial p}2 + G_2\frac{\partial y}{\partial p} + G_3\frac{\partial u}{\partial p} + \frac{\partial G}{\partial p}1 w_2 + \frac{\partial G}{\partial p}2 y + \frac{\partial G}{\partial p}3 u$$

which is an $(n-p)^{th}$ order system (requiring integration). Converting this into a block diagram yields the system of Fig.7.

4.2 SENSITIVITY RESULTS *

Before proceeding with examining the sensitivity plots it is important to check the curves, since they are useless unless they are giving the correct information. Also this gives a better understanding of their meaning. Fig. 8.6 represents a negative shift of 25% of the center of gravity from that in Fig. 8.1a. Concentrating, for instance on the overshoot peak of pitch angle, at about 1.2 sec. there is a

* Figs. 8.1-8.8 appear on pages 44-52 where Q refers to $\frac{\partial x}{\partial x}cg$, while R refers to $\frac{\partial x}{\partial \rho}$.
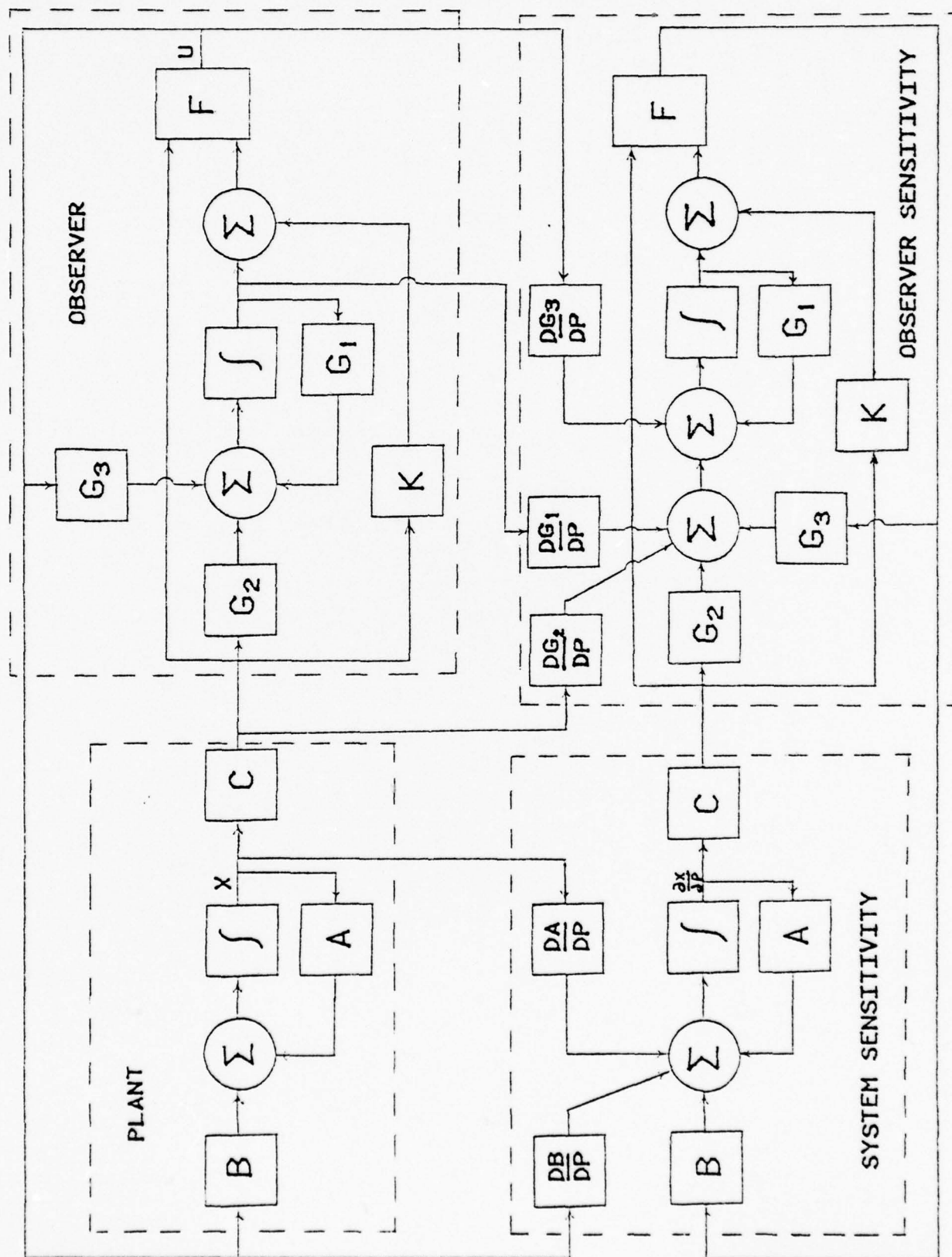
Figure 7. System Sensitivity With Observer Block Diagram

difference of about -.004. Now turning our attention to the corresponding sensitivity curve (Fig. 8.1b) of pitch angle at 1.2 sec. the value is around 0.016. When multiplied by the negative 25% shift yields -0.004 which checks with what was obtained above. The rest of the curves can similarly be checked in this manner for accuracy. The remainder of this chapter deals with interpreting the results of the sensitivity analysis.

4.2.1 Perturbation Effects

From Figs. 8.1a and 8.1b or Figs. 8.2a and 8.2b the trajectory for the pitch angle sensitivity is in phase with the trajectory for pitch angle itself. This can be seen by noting for instance that during approximately the first 1.5 seconds the sensitivity is positive while the pitch angle is also primarily positive ignoring the initial transition period. In other words if the center of gravity is shifted forward (in the positive direction) the effects of perturbation will be accentuated. After the first 3 seconds some oscillation is also introduced. Taking this into account it can be deduced that a positive shift of center of gravity will create a less stable situation. On the other hand velocity seems to yield a more favorable response for the same condition. This is concluded by noting that during rise time (the first 2.5 seconds) the trajectories for velocity and its corresponding sensitivity are in phase, while during the overshoot period the phases are opposite.

Thus a quicker rise time and less overshoot.

4.2.2 Disturbance Effects

From Figs. 8.3a and 8.3b the primary effect of shifting the center of gravity forward is to increase the overshoot for velocity but decrease it for pitch angle as is indicated by the plots using similar reasoning as above.

4.2.3 Set Point Effects

Examining Figs. 8.4a and 8.4b shows that a shift of center has very little effect on the response with respect to set point specification. This is determined by the relatively small magnitudes of the sensitivity trajectories. What little effect there is gives the pitch angle a quicker rise time (sensitivity positive in the first 2 seconds) and reduces the overshoot somewhat after that. There is no specific effect noticed on velocity.

4.2.4 Equilibrium Point Effects

The sensitivity does not seem to be significantly affected by operation at an equilibrium point other than the nominal, as is indicated by comparing the sensitivities of Figs. 8.1a and 8.6.

4.2.5 Observer Effects

The observer appears to have favorable effects on the response of both velocity and pitch angle. As can be seen from Figs. 8.1a and 8.1c, both velocity and pitch angle sensitivity are in phase with their corresponding state variable during rise time, while in the overshoot period they are of opposite phase. Thus the effect of the observer is to speed up rise time while reducing overshoot somewhat. The observer also seems to be somewhat more sensitive than without an observer, since the curves are of considerably larger magnitude than those without.

4.2.6 Air Density Sensitivity

Examining Figs. 8.1a and 8.8a or 8.2a and 8.8b indicates that for a higher (positive shift) air density the response to perturbation gets reduced. This is because when the state is positive the corresponding sensitivity is negative, tending to reduce the effect of the perturbation. The effects of disturbance inputs are different. From Figs. 8.3a and 8.8c the overshoot periods for both velocity and pitch angle seem to get lengthened (sensitivities are in phase and extend longer) for a positive shift of air density. For pitch angle the initial the initial overshoot magnitude gets decreased while for velocity it gets increased. Comparing the sensitivities with and without the observer (Figs.8.1a and 8.1d) the observer amplifies the effects of changing air density.
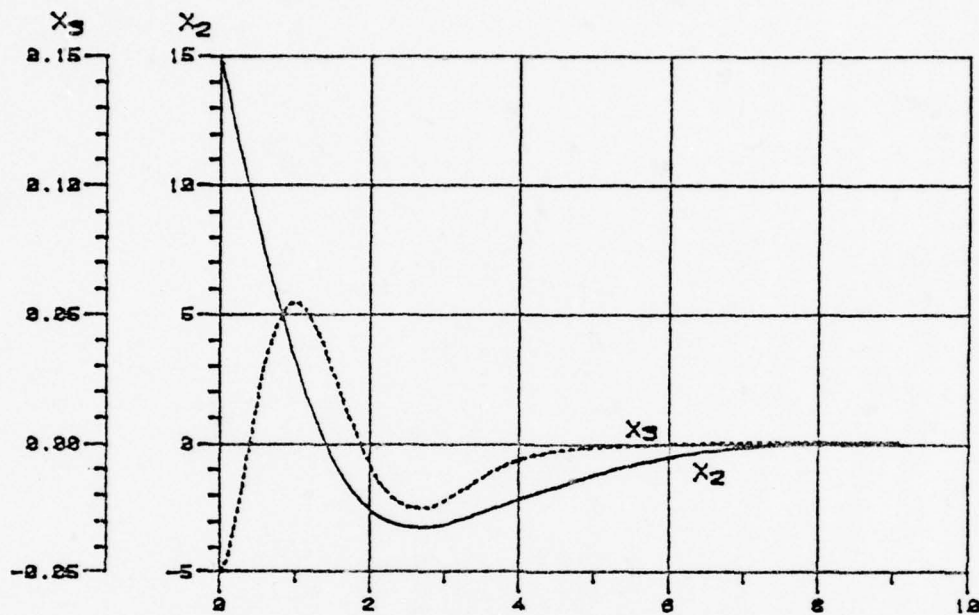
Figure 8.1a State Response of Linearized System
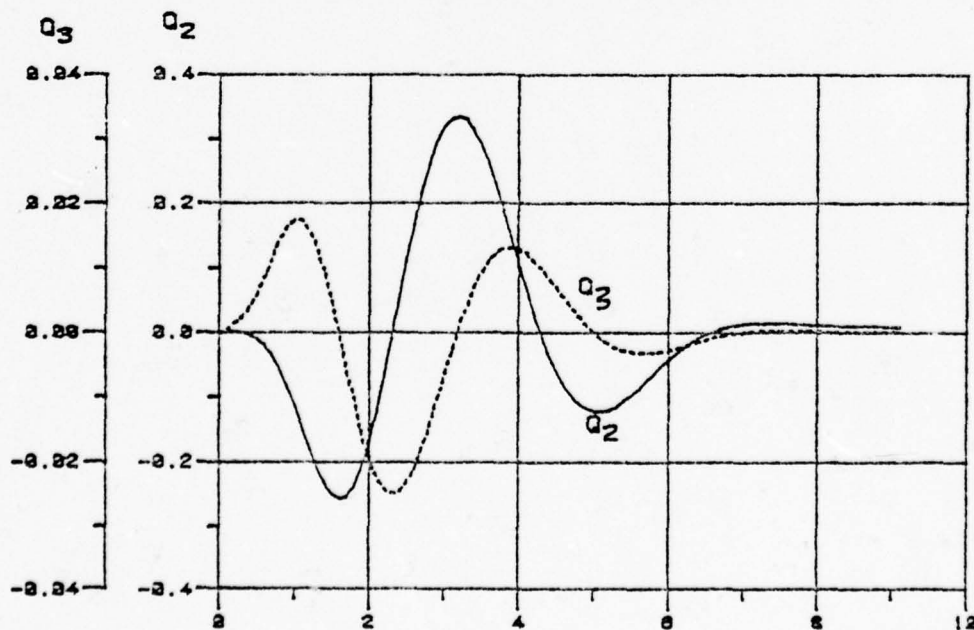$X_{2per} = 15.$ $X_{3per} = -0.05$

Figure 8.1b System Sensitivity with Respect to X_{os}
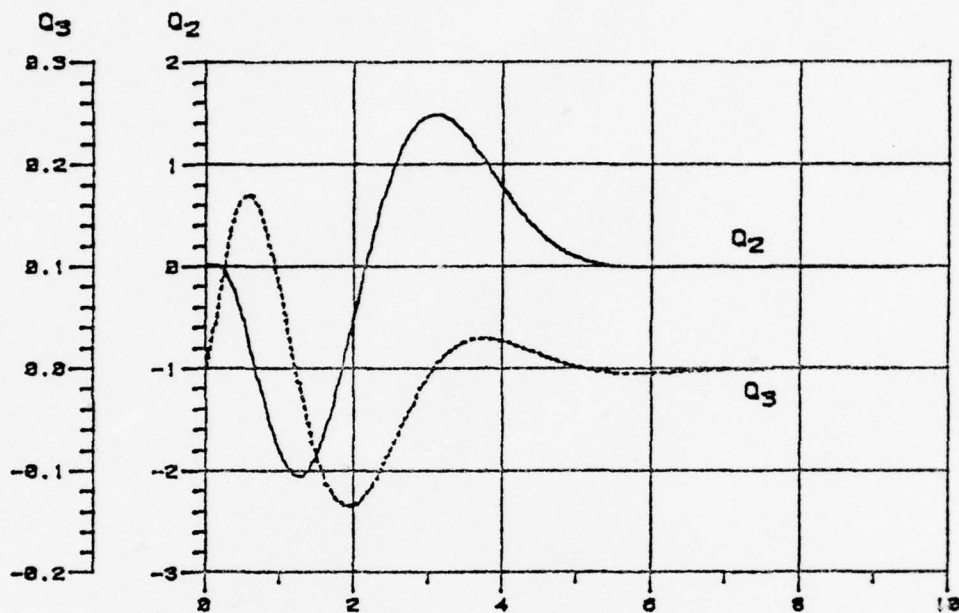$X_{2per} = 15.$ $X_{3per} = -0.05$

Figure 8.1c   System Sensitivity with Observer, with respect to X_{cg}
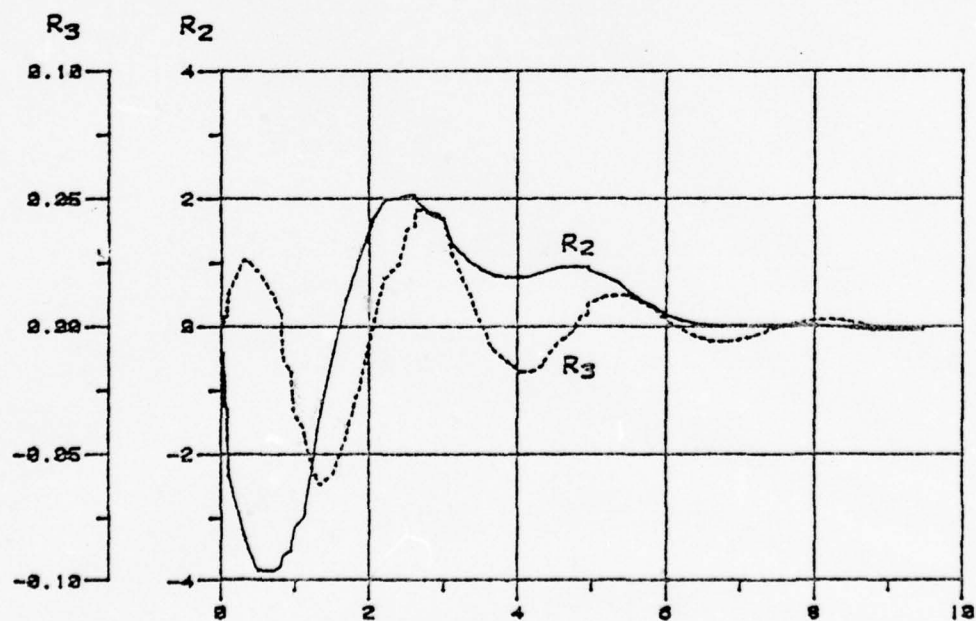X_{2per}= 15. X_{3per}= -0.05

Figure 8.1d   System Sensitivity with Observer, with respect to ρ
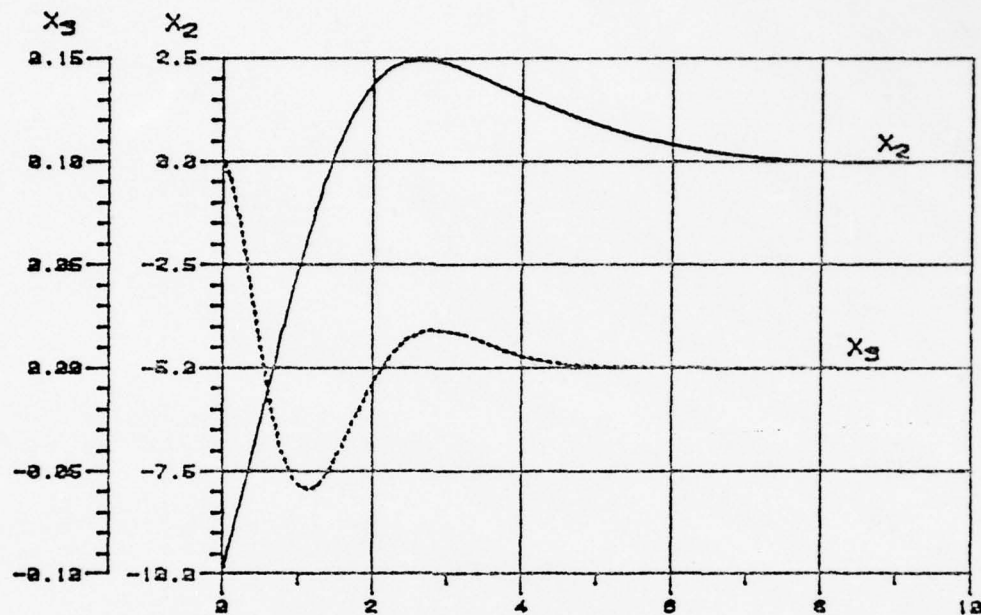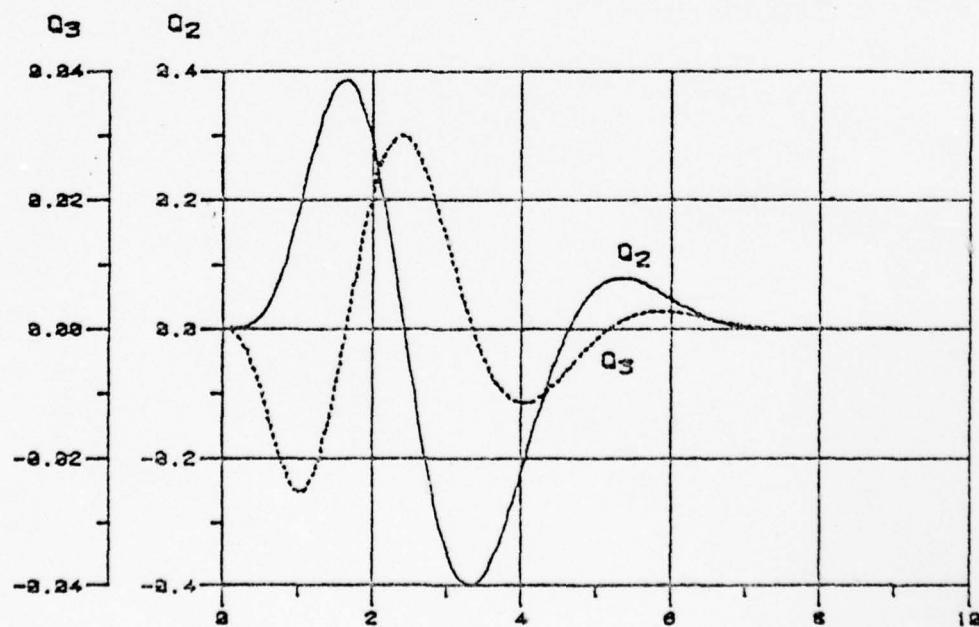X_{2per}= 15. X_{3per}= -0.05

Figure 8.2a State Response of Linearized System
$X_{2per} = -10.$ $X_{3per} = 0.1$

Figure 8.2b System Sensitivity with respect to X_{cg}
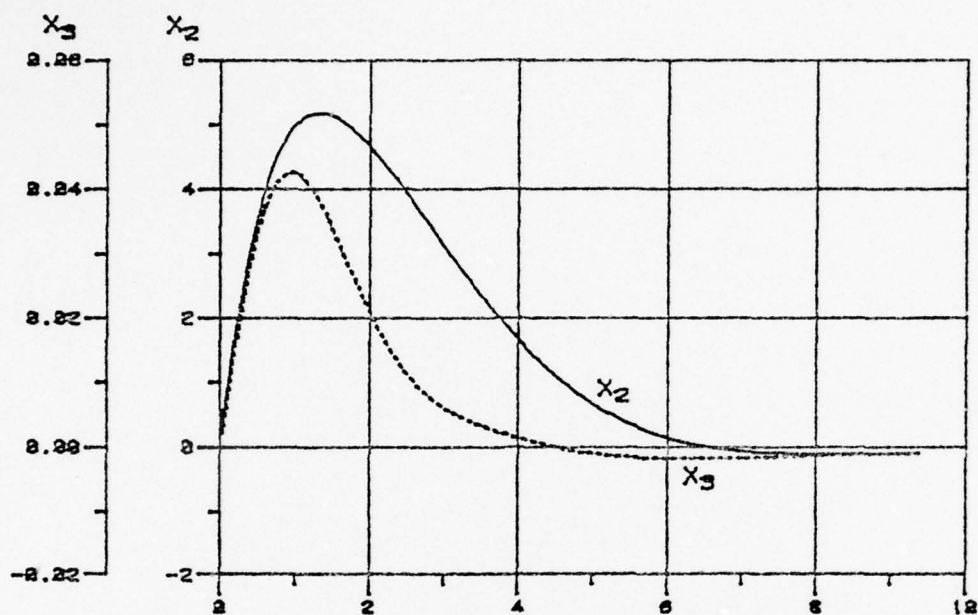$X_{2per} = -10.$ $X_{3per} = 0.1$

Figure 8.3a State Response of Linearized System
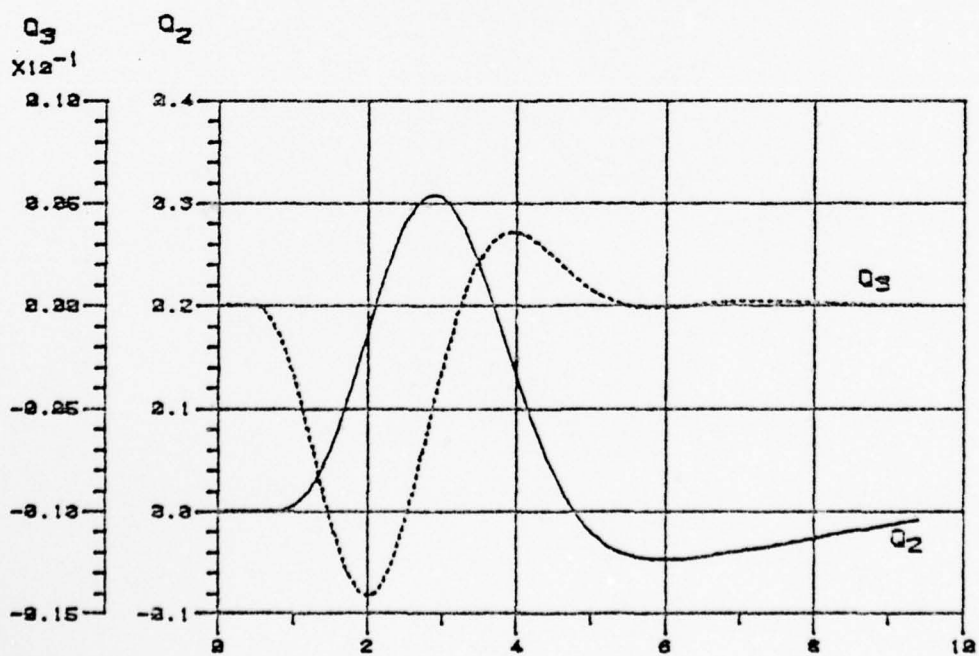$X_{2dis} = 10.$ $X_{3dis} = 0.08$

Figure 8.3b System Sensitivity with respect to X_{os}
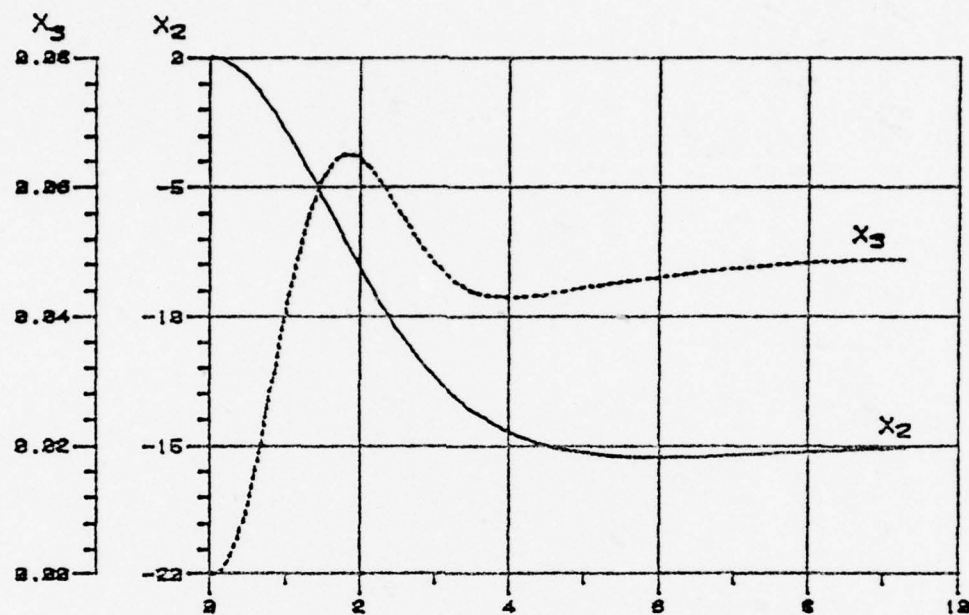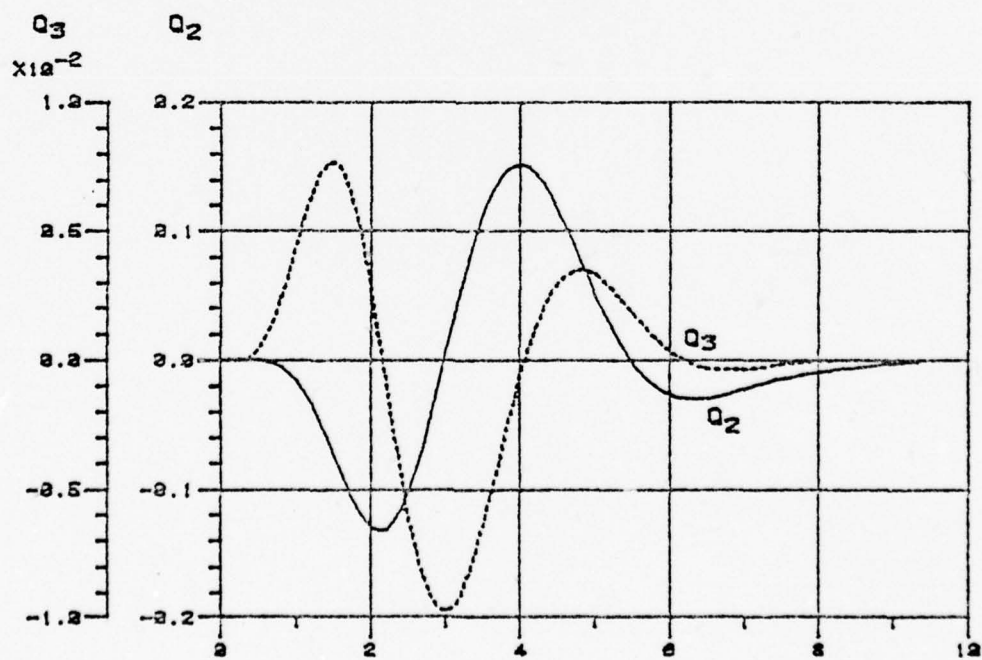$X_{2dis} = 10.$ $X_{3dis} = 0.08$

Figure 8.4a State Response of Linearized System
$X_{2set} = -15.$ $X_{3set} = 0.05$

Figure 8.4b System Sensitivity with respect to X_{os}
$X_{2set} = -15.$ $X_{3set} = 0.05$

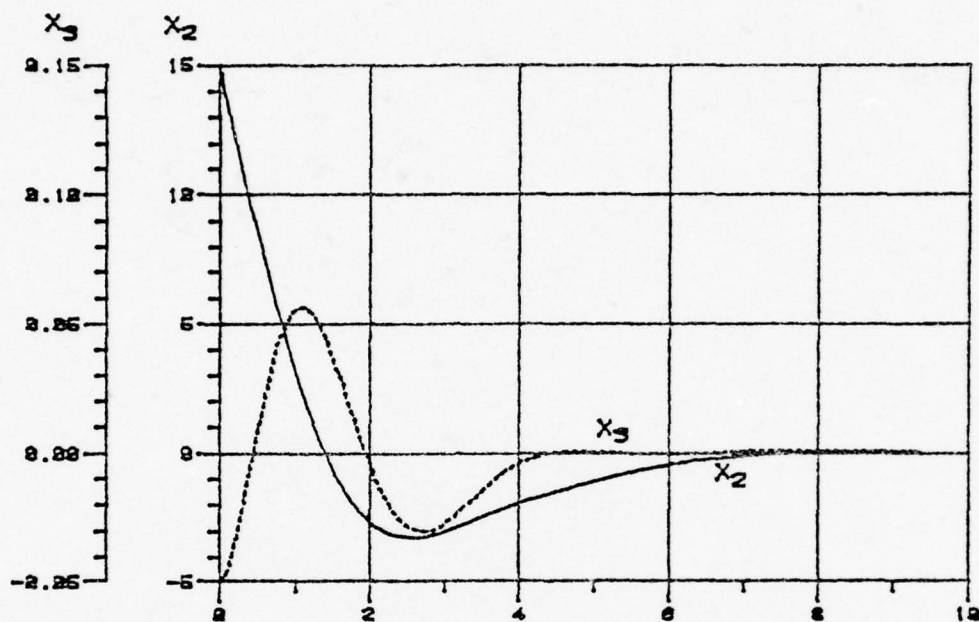Figure 8.5a   State Response of Linearized System, with Equilibrium Shift

X_{2per} = 15. X_{3per} = -0.05 X_{2e} = 170. X_{3e} = 0.06
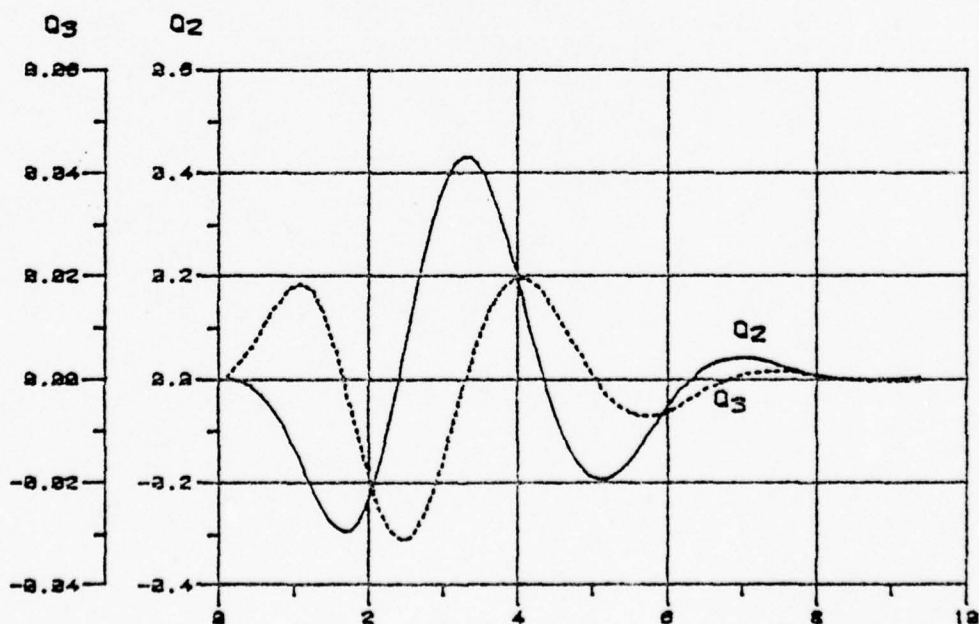
Figure 8.5b   System Sensitivity with respect to X_{ce}, with Equilibrium Shift

X_{2per} = 15. X_{3per} = -0.05 X_{2e} = 170. X_{3e} = 0.06
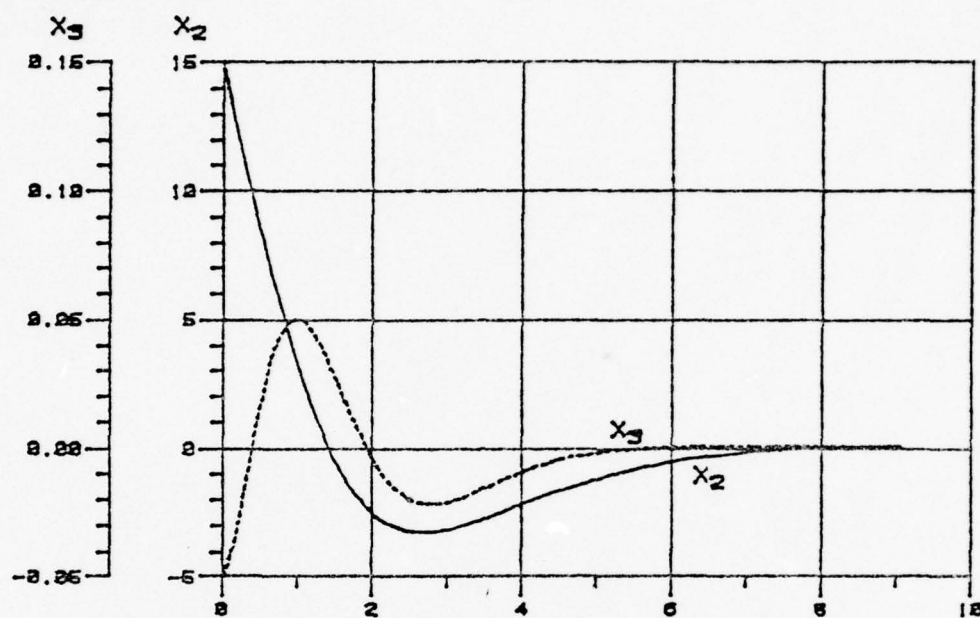
Figure 8.6   State Response of Linearized System, with
X_{co} Shifted -25%
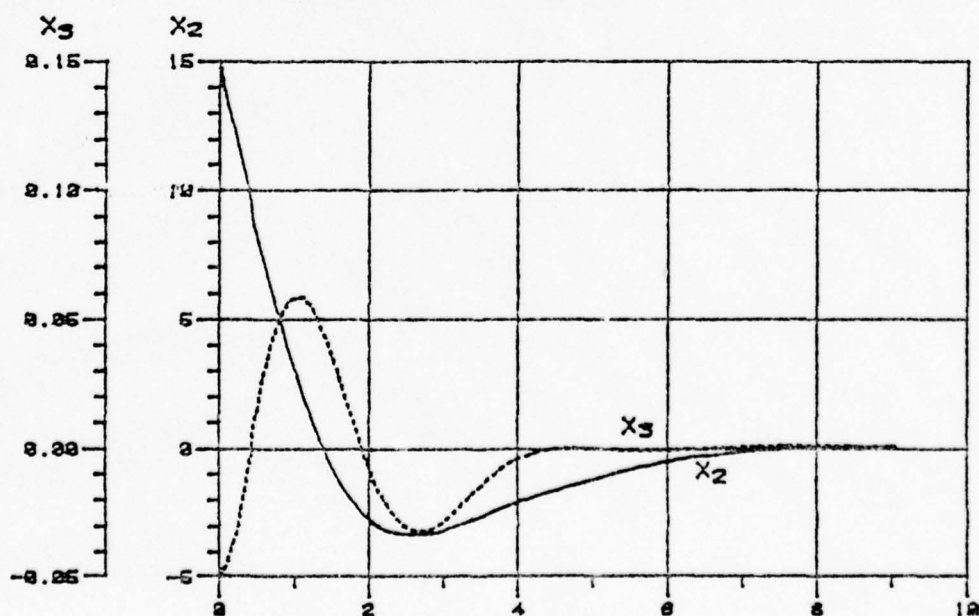
$X_{2per} = 15.$ $X_{3per} = -0.05$

Figure 8.7   State Response of Linearized System, with
ρ Shifted -25%

$X_{2per} = 15.$ $X_{3per} = -0.05$

Figure 8.8a   System Sensitivity with respect to ρ

X_{2per}= 15. X_{3per}=-.05

Figure 8.8b   System Sensitivity with respect to ρ

X_{2per}= -10. X_{3per}= 0.1

Figure 8.8c System Sensitivity with respect to ρ
X_{2dis}= 10. X_{3dis}= 0.08
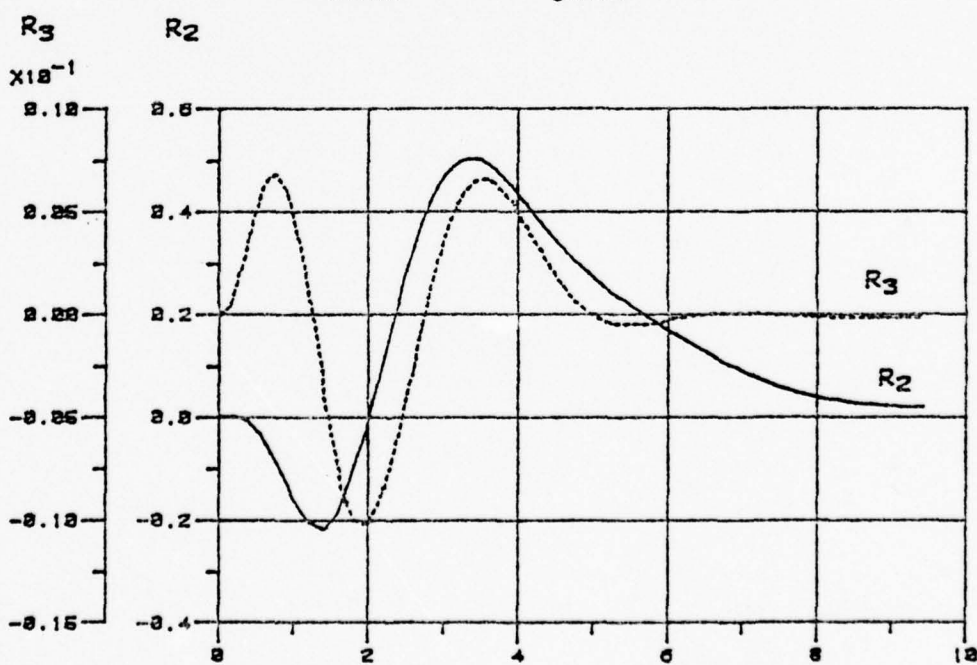
Figure 8.8d System Sensitivity with respect to ρ
X_{2set}= -15. X_{3set}= 0.05

CHAPTER 5

CONCLUSION

In this thesis just a few aspects of using optimal control theory as applied to aircraft flight have been examined. From this many desirable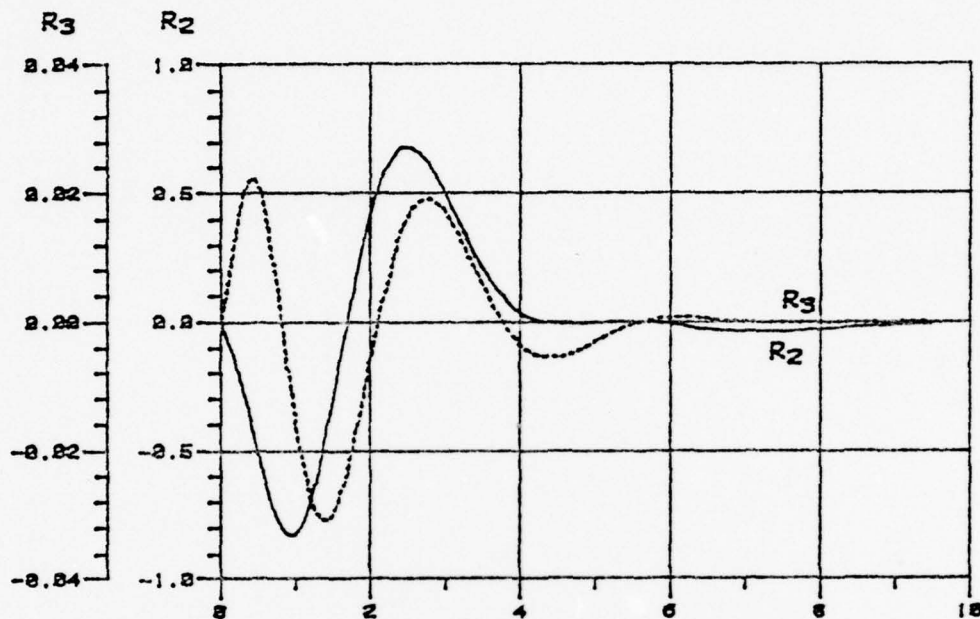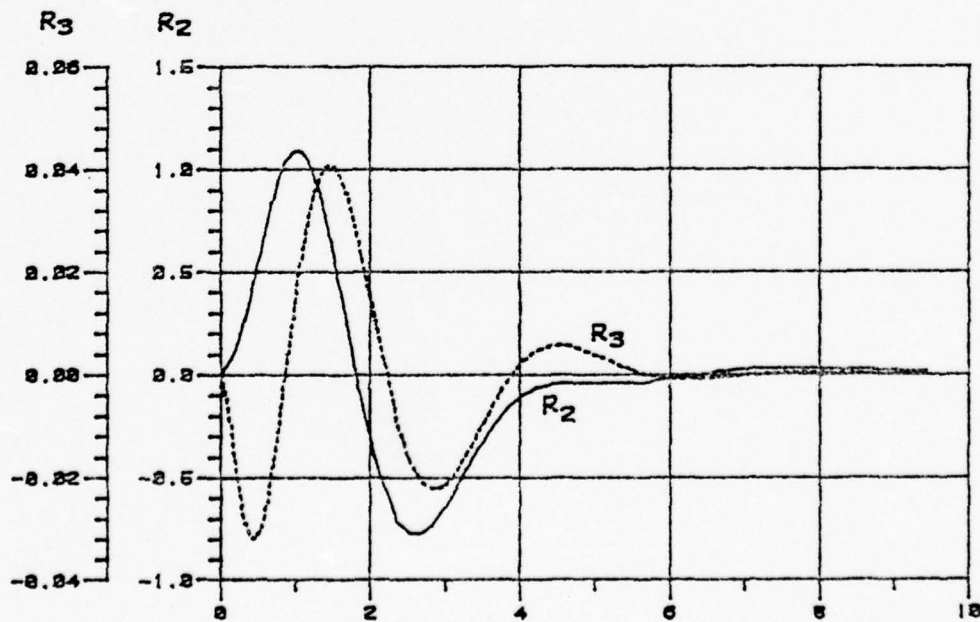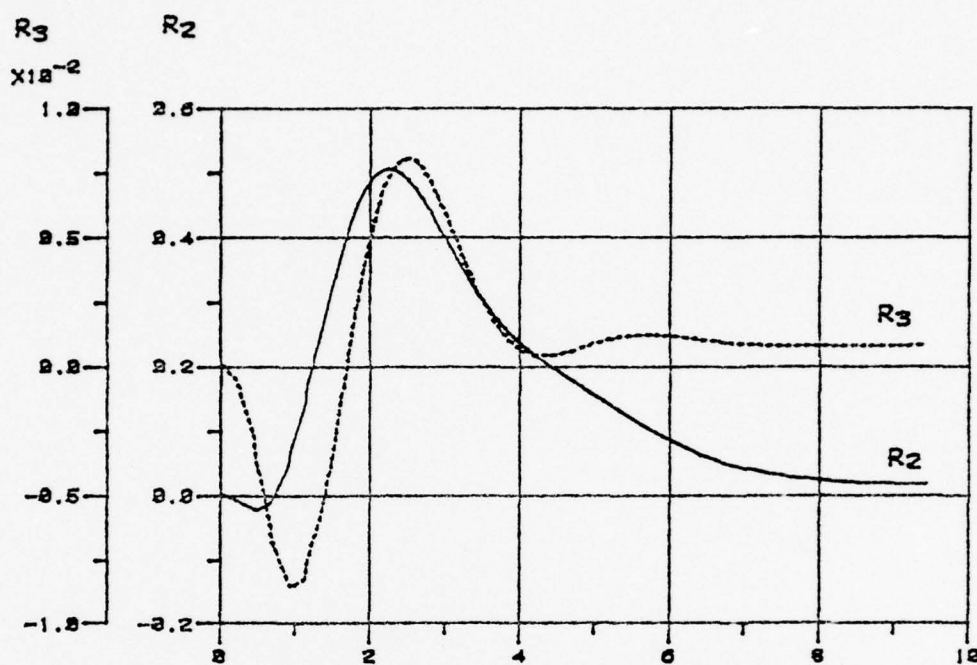 characteristics were revealed. But, on the other hand there were several shortcomings, which would have to be resolved before this method can be fully accepted and utilized.

A review of this control method shows some valuable features that should not be discounted. For example, controlling of the aircraft involves only simple command responses as opposed to a need for good knowledge of flight controlling, so pilot error can be greatly reduced or eliminated. Along these same lines near perfect accuracy can be attained. Furthermore a desired response can be chosen such as for maneuvarability or smoothness of flight, with automatic disturbance rejection.

Most of the drawbacks encountered had to do with flying outside of the nominal flight conditions. For instance, the plane would go unstable for a relatively small forward shift of center of gravity. This problem could be reduced by originally designing the controls around a more forward center of gravity, since the system stayed stable for large

backwards shifts of center of gravity. The problems of poor response characteristics and controls going out of bounds only occured at flight conditions well removed from the nominal and posed no problem while near the nominal flight conditions. Furthermore these problems could be alleviated by using a different feedback matrix or by incorporating the control bounds in the controller itself.

Probably the most serious deficiencies to this method is the unfavorable response characteristics at flight conditions away from the nominal. The technique most widely used, currently, to bypass this problem is to have many precalculated feedback gain matrices available, and to just use one in accordance with the flight condition the matrix most closely corresponds to. This is a practical solution but seems somewhat awkward. A more recent, and perhaps more promising concept involves the use of adaptive control. This method automatically adjusts the gain matrices in accordance with the changing flight conditions. This type of control has been applied to vertical takeoff and landing (VTOL) aircraft with favorable results and quite possibly is the method of the future.

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APPENDIX A

DESIGN OF A LOW ORDER OBSERVER.

Given a system of the form:

$$\dot{x} = Ax + Bu \qquad\qquad A[n \times n] \qquad B[n \times m]$$

$$y = Cx \qquad\qquad\qquad C[p \times n]$$

where (A,C) is observable and C is of full rank (Ref. 4). Choose a transformation matrix $T^T = [C \; M]$ where M is an $(n-p) \times n$ matrix, such that T is nonsingular (often chosen so that T simply interchanges the order of the state variables x_i (i.e. $[\; x_2, x_1, x_4, \ldots]$). Using this transformation:

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = Tx = \begin{pmatrix} y \\ z_2 \end{pmatrix}$$

$$\dot{z} = TAT^{-1}z + TBu$$

$$A = TAT^{-1} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$$B = TB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

1) $\qquad \dot{z}_1 = A_{11}z_1 + A_{12}z_2 + B_1u$

2) $\dot{z}_2 = A_{21}z_1 + A_{22}z_2 + B_2u$

Since $z_1 = y$ is already known, we only need an observer for z_2. Now define:

3) $\dot{\hat{z}}_2 = A_{22}z_2 + (A_{21}z_1 + B_2u) + K(A_{12}z_2 - A_{12}z_2)$

and call $e = \hat{z}_2 - z_2$

4) $\frac{de}{dt} = \frac{d}{dt}(\hat{z}_2 - z_2) = (A_{22} - KA_{12})(\hat{z}_2 - z_2)$

The eigenvalues of this can be placed such that the error, $e = 0$ in the steady state. This is possible since (A,C) is observable which implies (A_{22}, A_{12}) is controllable.

From 1) and 3) we see that:

$$\dot{\hat{z}}_2 = A_{22}z_2 + (A_{21}z_1 + B_2u) + K(z_1 - A_{11}z_1$$
$$- B_1u - A_{12}\hat{z}_2) + (A_{22} - KA_{12})(Kz_1)$$

$$\frac{d}{dt}(\hat{z}_2 - Kz_1) = (A_{22} - KA_{12})(\hat{z}_2 - Kz_1) + (A_{21} - ka_{11}$$
$$+ (A_{22} - KA_{12})K)z_1 + (B_2 - KB_1)u$$

let

$$w_2 = \hat{z}_2 - Kz_1$$

then

$$\dot{w}_2 = G_1w_2 + G_2y + G_3u$$

where:

$$G_1 = A_{22} - KA_{12}$$

$$G_2 = A_{21} - KA_{11} + G_1K$$

$$G_3 = B_2 - KB_1$$

This yields an observer system as shown in Fig.9, and x may now be obtained from the equation:

$$x = T^{-1}\begin{bmatrix} y \\ z_2 \end{bmatrix}$$

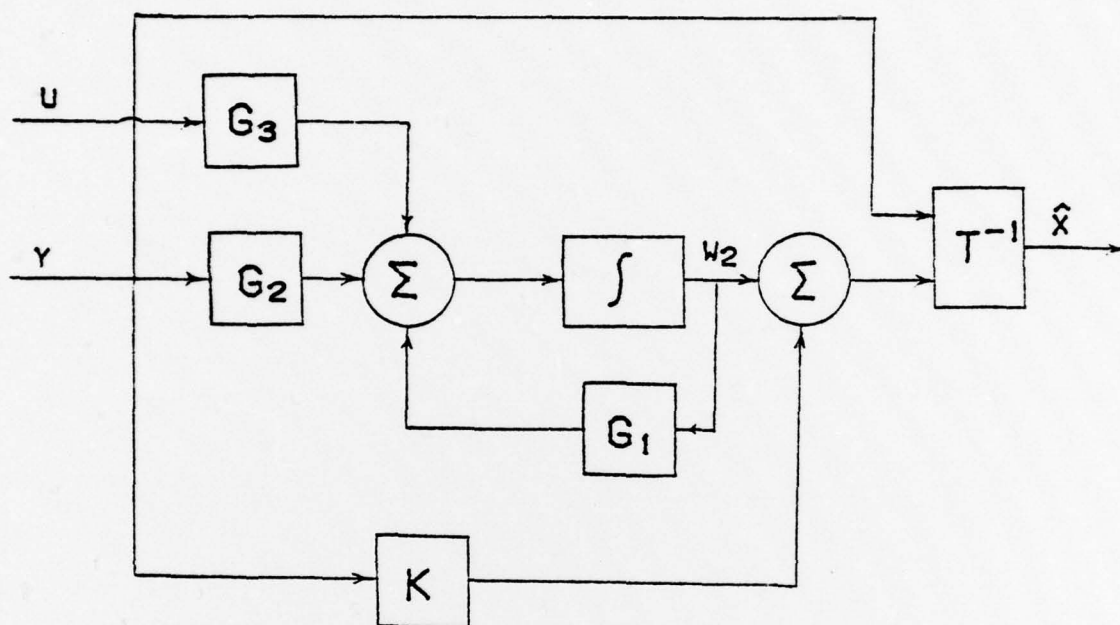Figure 9.   Linear Observer Block Diagram

APPENDIX B

MINIMUM PRINCIPLE

Given a system of the form :

$\dot{x} = Ax + bu$

$y = Cx$

and a performance index:

$J = (1/2)\int_0^\infty (x^T Q x + u^T R u)dt$

Where Q and R are positive semidefinite and positive definite, symmetric cost matrices. Using the techniques as discussed in Athans and Falb (Ref. 1), form the Hamiltonian:

$H = L + p^T \dot{x} = (1/2)(x^T Q x + u^T R u) + p^T(Ax + Bu)$

From the Minimum Principle, for unconstrained u (hopefully u will stay within constraints):

$0 = \frac{\partial H}{\partial u} = u^T R + p^T B$

$u = R^{-1} B^T p$

$\dot{p} = -\frac{\partial H}{\partial x} = -Qx - Ap^T$

If we now assume p = Kx (as discussed in Athans and Falb) where K is time invariant since A,B,Q, and R are all time invariant, we find:

$$u = R^{-1}B^TKx$$

From the costate,p, equation,then

$$KA + A^TK + KBR^{-1}B^TK + Q = 0$$

Which can be solved for a unique K,so

$$u = Fx \quad \text{where } F=R^{-1}B^Tp$$

APPENDIX C
COMPUTER PROGRAMMING

To get analytical results for all of the equations just derived computer programs had to be written to perform all of the integrations and other related operations needed. For reasons of time and accuracy, all preliminary simulation was done on the DEC 10 digital computer. Final results were then tested on the hybrid computer, where the AD-5 analog computer represented the real time plant (an airplane) which was being controlled by the digital PDP-11 digital computer.

1.DEC-10

All control testing and sensitivity analysis was done on the DEC-10. Because of the size of the program and the need for versatility of input data, an interactive format was utilized. This method, of having the operator respond to different options (e.g. initial conditions),helped facilitate debugging of the program also. Furthermore this method made it possible to study any flight condition or

possibly any aircraft configuration within reason, by a simple response to a parameter change option. The only true shortcoming involved here was that the program did not have the option of generating feedback matrices (this was done on Linsys, using the Riccati package) so the responses to different conditions (other than the initially chosen one) were suboptimal in some sense.

All of the interactive programming and condition organization was done with one very large main program, ACUTI. This program would ask for the desired flight conditions or the desired data analysis (such as sensitivity or observer analysis) and would then make calls to the various subprograms needed to facilitate these. The subprograms would then execute the different commands such as for integrations or plots.

2.HYBRID

The AD-5 analog computer had the nonlinear aircraft plant equations patched into it, thus simulating a real time airplane. This required a lot of manipulation and scaling due to the limited amount of hardware available, and due to saturation restrictions. To help set up and test this, several PDP-11 programs were used. Again, here, the programs were set up interactively, so theoretically, any

flight conditions could be simulated. But here, again, due to scaling and hardware limitations, there was actually only a limited range of variations possible. For accuracy and speed of setting up, a subroutine, POTREV, was used to calculate and set all pot values, automatically, according to what parameters were desired. The analog diagram is shown in Fig.10.
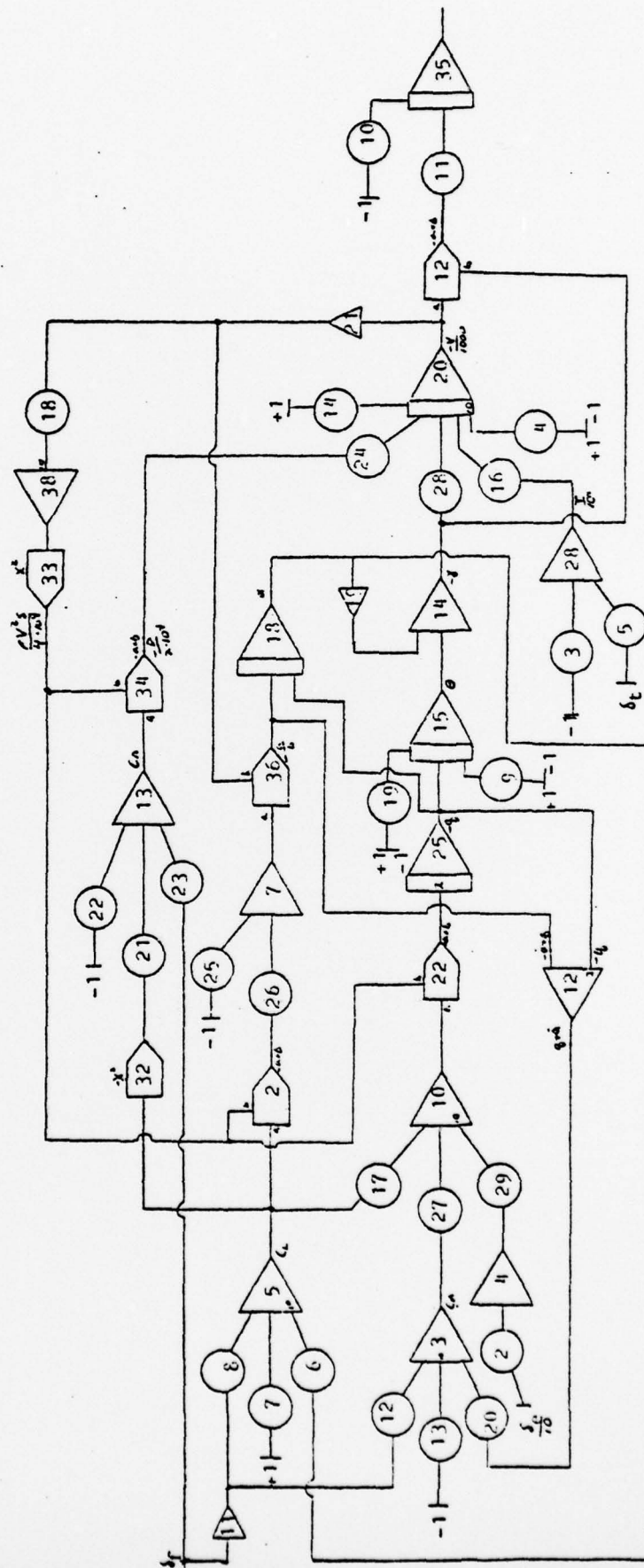
Figure 10. Analog Block Diagram

APPENDIX D

LINEARIZED MATRICES

```
        A
-3.09 -0.18  0.00  1.00  0.00  0.00  0.00
 0.14 -0.07 -0.32  0.00 -0.00  0.00  0.00
 0.00  0.00  0.00  1.00  0.00  0.00  0.00
-0.74  0.09  0.00 -1.01  0.00  0.00  0.00
-1.91  0.00  1.91  0.00  0.00  0.00  0.00
 0.00  1.00  0.00  0.00  0.00  0.00  0.00
 0.00  0.00  1.00  0.00  0.00  0.00  0.00

        B
 0.00 -0.24  0.00
 0.00 -0.04 -0.16
 0.00  0.00  0.00
-1.37 -1.48  0.00
 0.00  0.00  0.00
 0.00  0.00  0.00
 0.00  0.00  0.00

        C
 0.05  0.00  0.00  0.00  0.00  0.00  0.00
 0.00  1.20  0.00  0.00  0.00  0.00  0.00
 0.00  0.00  0.05  0.00  0.00  0.00  0.00
 0.00  0.00  0.00  0.10  0.00  0.00  0.00
 0.00  0.00  0.00  0.00  0.25  0.00  0.00
 0.00  0.00  0.00  0.00  0.00  2.50  0.00
 0.00  0.00  0.00  0.00  0.00  0.00  2.50

        R
10.00  0.00  0.00
 0.00  0.14  0.00
 0.00  0.00  2.00
```

```
        FR
-0.60 -3.16   7.81   2.59   0.24 -2.20   7.22
 0.05  0.09   0.05   0.04 -0.09  0.05   0.31
 0.20  6.14  -0.56 -0.07   0.05   3.39   0.87

        RR
-2.62 -2.62  -1.18 -1.18 -0.57 -0.57 -0.01

        RI
 0.38 -0.38   1.88 -1.88   0.49 -0.49   0.00


        K
72.49 -0.74   0.00   0.00   0.00
 0.07998.99   0.00   0.00   0.00
```